NP

## If both x and y are odd, is xy odd?

g



Odds & Evens

#### Answer: Yes, xy is odd

Odd numbers can be represented as 2m + 1 or 2n + 1, where *m* and *n* are integers. (Think about why this is.) Multiplying two numbers of this form together would yield 4nm + 2m + 2n + 1, which is always odd; the 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> terms are multiplied by 2 (or 4), so they are even, as is their sum. An even number plus one is odd. Thus *xy* is odd.

More simply, we could just recall: an odd number times an odd number is always odd.

When in doubt, try it out! Pick numbers to test properties.

ManhattanGMAT Number Properties Strategy Guide Odds & Evens Copyright © 2009 MG Prep, Inc. NP

Positives & Negatives

#### **Answer: Sufficient**

ls x < 0?

1)  $xy^2 < 0$ 

Any number, except for 0, raised to an even power will be positive. If y were 0, the inequality would not be true, so we know that  $y^2$ , regardless of the sign of y, will be positive. For  $xy^2$  to be less than zero, that means that x must be negative. The statement is sufficient.

Is the statement sufficient?



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## Simplify $\sqrt{6,300}$ .

g



#### Simplifying a Root

Answer:  $30\sqrt{7}$ 

Whenever simplifying an expression under the square root sign, factor the expression. In this case,  $6,300 = 2^2 \times 3^2 \times 5^2 \times 7$ . For every pair under the square root sign, move one outside the radical, and throw the other away:  $\sqrt{2^2 3^2 5^2 7}$ becomes  $(2)(3)(5)\sqrt{7}$ , or simply  $30\sqrt{7}$ .



3



If both x and y are odd, is  $x^2 + y$  odd?

## g

## Odds & Evens

#### Answer: No, $x^2 + y$ is even

Odd numbers can be represented as 2m + 1 or 2n + 1, where *m* and *n* are integers. (Think about why this is.)  $(2m + 1)^2 = 4m^2 + 4m + 1$ , and adding 2n + 1 would yield  $4m^2 + 4m + 2n + 2$ . This is always even, since a 2 can be factored from all four terms.

More simply, we could just recall: an odd number times an odd number is always odd, and an odd plus an odd is always even.

When in doubt, try it out! Pick numbers to test properties.



NP

ManhattanGMAT Number Properties Strategy Guide Odds & Evens

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If x is odd and y is even, is xy odd or even?



Odds & Evens

#### Answer: xy is even

An odd number can be represented as 2m + 1, and an even number can be represented as 2n, where *m* and *n* are integers. (Think about why this is.) Multiplying 2m + 1 and 2n would yield 4mn + 2n, which is always even, since 2 is a factor of both terms. (Factor out the 2 to get 2(2mn + n), which shows that this number will be even.)

More simply, we could just recall: an odd number times an even number is always even.

When in doubt, try it out! Pick numbers to test properties.



ManhattanGMAT Number Properties Strategy Guide

5



Is the statement sufficient?

ls *x* < 0?

1)  $\sqrt[13]{x} < 0$ 

### Odd Roots

#### **Answer: Sufficient**

Don't let the 13 confuse you; the only thing that matters is that 13 is an odd number. Odd roots, as well as odd exponents, preserve the sign of the number inside. If  $\sqrt[13]{x} < 0$ , then x is also less than 0. The statement is sufficient.



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## If both x and y are odd, is x - y odd?

## g



### Simplifying a Root

Answer:  $17\sqrt{5}$ 

When a square root lurks in the denominator, we can rationalize the denominator by multiplying by the appropriate

form of 1 – in this case,  $\frac{\sqrt{5}}{\sqrt{5}} \cdot \left(\frac{85}{\sqrt{5}}\right) \left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{85\sqrt{5}}{5}$ , and 85

divided by 5 is 17, so the simplest form is  $17\sqrt{5}$ .

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7



### Odds & Evens

#### Answer: No, x - y is even

Odd numbers can be represented as 2m + 1 or 2n + 1, where m and n are integers. (Think about why this is.) Subtracting two numbers of this form would yield (2n + 1) - (2m + 1), or just 2n - 2m, which is always even, since a 2 can be factored out of the remaining terms (i.e., 2(n - m)).

More simply, we could just recall: an odd number minus an odd number is always even.

When in doubt, try it out! Pick numbers to test properties.



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g

## NP

*x* is divisible by 42. Which of the following numbers is definitely a factor of  $x^2$ ? (Choose all that apply.)





### Exponents

Answer: –1

Since  $(-1) \times (-1) = 1$ , -1 raised to any even power is 1. If you multiply by -1 one more time, you end up with -1, so -1 raised to any odd power will equal -1. 789 is an odd number, so  $(-1)^{789} = -1$ .

9



## Divisibility

#### Answer: a) 63 and c) 36

If x definitely has 2, 3 and 7 as factors, then when we square x, we know that  $x^2$  will have two 2s, two 3s and two 7s as factors. 63 is  $7 \times 3 \times 3$ , and 36 is  $2 \times 2 \times 3 \times 3$ . Using the factor foundation rule, we can guarantee that all numbers that solely use those factors are factors of  $x^2$ . Both 63 and 36 use only prime factors found in  $x^2$ .





 $\left(\sqrt[5]{n}\right)^{5}$  is always equal to which of the following?

a) *n* b) *n*<sup>25</sup> c) *n*<sup>1/5</sup>

c) *n* d) 1

# g

## NP

### Exponents & Roots

Answer: a) n

Try it:  $(\sqrt[5]{n})^5$  is the same as  $(n^5)^{\frac{1}{5}}$ , which is equal to *n*, since  $(n^a)^b = n^{ab}$ , and 5 times 1/5 equals 1.

Alternatively,  $(\sqrt[5]{n})^5 = (\sqrt[5]{n})(\sqrt[5]{n})(\sqrt[5]{n})(\sqrt[5]{n})(\sqrt[5]{n}) = n.$ 

You can try this out if you need convincing. Pick a few numbers and see what happens!



11



If both x and y are even, is x - y even?

g

## Odds & Evens

#### Answer: Yes, x - y is even

Even numbers can be represented as 2m or 2n, where m and n are integers. (Think about why this is.) Subtracting two numbers of this form would yield 2n - 2m, or 2(n - m), which has 2 as a factor, so it is even.

More simply, we could just recall: an even number minus an even number is always even.

When in doubt, try it out! Pick numbers to test properties.



NP

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### Is the statement sufficient?

*xy* < 0. Is *y* < 0?

1)  $y^2 \sqrt{x} > 0$ 

# g



If the units digit of an integer is 7, then which one-digit integers is it definitely NOT divisible by?

## g



Even Roots

#### **Answer: Sufficient**

If we know that xy < 0, then we know that x and y have different signs – one must be positive and the other negative. From the statement, we know that x must be positive, because we are not allowed to take an even root of a negative number. If x is positive, then y must be negative. The answer to the question is yes, and the statement is sufficient.





### Divisibility Rules

#### Answer: 2, 4, 5, 6, and 8

Integers that are divisible by 2, 4, 6, or 8 end in 2, 4, 6, 8, or 0; those divisible by 5 end in 5 or 0.

As an exercise, try to provide examples of integers with a ones digit of 7 that are divisible by 1, 3, 7, and 9.



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## (NP)

Is the statement sufficient?

If *x* is divisible by *y*, is *x/y* odd?

1) x and y are both odd.

# g



### Fractional Exponents

Answer: 32

Using the rules of exponents,  $16^{\frac{5}{4}} = (16^5)^{\frac{1}{4}} = (16^{\frac{1}{4}})^5$ .

Since it is easier to calculate  $16^{\frac{1}{4}}$  than it is to calculate  $16^5$ , the latter representation will be easier to simplify.  $16^{\frac{1}{4}} = 2$ , and  $2^5 = 32$ .

15



### Odds and Evens

#### Answer: Sufficient

This question is tricky, because an odd divided by an odd can yield an odd integer *or a non-integer*. However, the question stem states that x is divisible by y. Therefore, x/y is an integer, and the result must be odd.



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If an integer that is divisible by 6 is squared, then which (nonzero) onedigit integers is this squared result definitely divisible by?

g



Answer: 1, 2, 3, 4, 6, and 9

Call the original integer *n*. Since *n* is divisible by 6, we can say n = 6m, where *m* is any integer. Squaring *n* yields  $n^2 = (6m)^2 = 36m^2$ . Since 36 is divisible by 1, 2, 3, 4, 6, and 9, they are all factors of  $n^2$  as well.

Any combination of 5, 7, and/or 8 may also divide  $n^2$ , but we can't say for sure whether they do without knowing what *m* is.

17





## Divisibility |

### Simplifying Exponential Expressions

#### Answer: 5<sup>3</sup>

Instead of multiplying out everything, look for ways to reduce. On the top of the fraction,  $6^4$  can be separated into  $(2^4)(3^4)$ . This can be cancelled with the  $2^4$  and  $3^4$  on the bottom of the fraction, so we are left with  $\frac{50^3}{10^3}$ , which can be reduced to  $5^3$ . (Note that  $5^3 = 125$ .)



NP

## Simplify the following expression:

 $(4(6(8(9^0))^1)^{-1})^2$ 

g



If the ones digit of an integer is 0, then which (nonzero) one-digit integers is the integer definitely NOT divisible by?

g



#### Order of Operations

Answer: 1/144

PEMDAS dictates the order of operations to perform. We must always calculate the innermost parentheses first, then work our way outwards. Calculate  $9^0 = 1$  first; then  $8^1 = 8$ . Next we have  $(6(8))^{-1} = 1/48$ .; then  $(4/48)^2 = 1/144$ .

It's easy to remember PEMDAS with this saying: Please Excuse My Dear Aunt Sally!



19



## Divisibility Rules

#### Answer: None

It could be divisible by <u>any</u> of the one-digit integers! (Except for 0; dividing by 0 is always off limits.)

To verify, take any nonzero one-digit integer, multiply it by ten, and the product will end in zero and be divisible by the original one-digit integer.



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## If x is even and y is odd, is $x^{2} + y^{2}$ even or odd?



Odds & Evens

#### Answer: $x^2 + y^2$ is odd

An even number can be represented as 2m, and an odd number can be represented as 2n + 1, where *m* and *n* are integers. Squaring the even number yields  $4m^2$ ; the odd,  $4n^2 + 4n + 1$ . Adding these together yields  $4m^2 + 4n^2 + 4n + 1$ . The 1<sup>st</sup> 3 terms all have 4 as a factor, so their sum is even, and an even number plus 1 is odd.

More simply, we could just recall: an even number squared is always even, an odd number squared is always odd, and an even plus an odd is always odd.

When in doubt, try it out! Pick numbers to test properties.



21



Given that  $y^7 < y^6$ , describe all of the possible values for y.



### Testing Positive and Negative Cases

Answer: y < 1, but not equal to 0 (alternatively, 0 < y < 1or y < 0)

Think about various categories of numbers: if y were negative, then  $\gamma^7$  would also be negative, while  $\gamma^6$  would be positive; then  $y^7 < y^6$ . If y = 0 or 1, then  $y^7 = y^6$ , which is not acceptable. When *y* is between 0 and 1,  $y^7 < y^6$ , since  $y^7$  would equal  $y^6$ times some fraction between 0 and 1. Finally, when y > 1,  $y^7 > 1$  $y^6$ .



NP



## Is the statement sufficient?

The positive integer x is a prime number. What is x?

1) x + 11 is a prime number.

g



Prime Numbers

#### **Answer: Sufficient**

If you tested numbers to answer this question, you probably figured out pretty quickly that 2 is a possible value of x. If you continue to test numbers to make sure there are no other possible values for x, you may notice a pattern emerging. 11 + 3 = 14, 11 + 5 = 16, 11 + 7 = 18, etc. 11 plus any prime besides 2 will yield an even number. 2 is the only even prime, because every other even number has 2 as a factor. Therefore, x must equal 2. The statement is sufficient.



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23



Is the statement sufficient?

ls *a* < 0?

1)  $a^{b} < 0$ 

g

#### Exponents

#### **Answer: Sufficient**

In order for  $a^{\flat} < 0$ , *a* must be negative. (This is equivalent to saying that a < 0.)

If *a* were nonnegative, then the minimum value  $a^b$  could take would be 0, regardless of the value of *b*.



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### Is the statement sufficient?

Is x/y even?

1) x and y are both even.

g



What is the greatest number of primes that could be included in a set composed of four consecutive integers? Name the elements of the set.

## g



Odds & Evens

#### Answer: Insufficient

Even numbers can be represented as 2m or 2n, where m and n are integers. (Think about why this is.) Dividing would give (2n)/(2m), or just n/m. Not only can it not be determined whether this result is even (e.g., x = 40 and y = 4) or odd (e.g., x = 44 and y = 4), we cannot even determine that it will be an integer! (e.g., x = 42 and y = 4.) The statement is insufficient.

NP

### Prime Numbers

#### Answer: 3, in the set {2, 3, 4, 5}

Any set composed of four consecutive integers will contain two even and two odd integers. Since 2 is the only even integer that is prime, no such sets can have four primes, and sets that do not contain 2 can have, at most, two primes. The only set with three primes is {2, 3, 4, 5}.

Why isn't {1, 2, 3, 4} acceptable as another solution to this question?



25

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## What is the greatest common factor of 990 and 924?



## NP

x is divisible by 144. If  $\sqrt[3]{x}$  is an integer, then which of the following is  $\sqrt[3]{x}$  definitely divisible by? (Choose all that apply) 144

a) 4 b) 8 c) 9 d) 12

3, 3

## NP

Greatest Common Factor

Answer: 66

To find the Greatest Common Factor of 2 or more numbers, you need to figure out all the factors they share in common. In this case, 990 and 924 each have one 2, one 3 and one 11. That means that the GCF will be  $2 \times 3 \times 11$ , or 66.

27

## NP

#### Answer: a) 4 and d) 12

Remember that when we complete a prime box for a variable, that variable could still have additional factors. For the cube root of a number to be an integer, the original number must have 3 of each prime factor, or some multiple of 3 (3, 6, 9, etc.). In this case, that means the factors of x that we can't see <u>must</u> include at least two additional 2s and one additional 3. From this information, we can definitively conclude that  $\sqrt[3]{x}$  must have two 2s and one 3 as factors. 4 and 12 are the only numbers in the list we can guarantee are factors of  $\sqrt[3]{x}$ .



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## Divisibility

What is the only two-digit number that is both a perfect square and a perfect cube?



Exponents & Roots

#### Answer: 64

We need a 2-digit integer that is both a perfect square and a perfect cube. This set includes all integers of the form  $m^3 = n^2$ , where both m and n are integers. Manipulating the equation tells us that  $n = m^{3/2}$ . Thus we can only choose integers for m that will make *n* an integer—so *m* must be a perfect square. The only perfect square that works is 4:  $4^3 = 64$ , a 2-digit integer. 9 doesn't work, because  $9^3 = 729$ , a 3-digit integer. 1 doesn't work either, because  $1^3 = 1$ , a 1-digit integer.



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29

Is the statement sufficient?

 $|\mathbf{s}| |x| > |y|?$ 

1) x - y > 0



Testing Positive and Negative Cases

#### Answer: Insufficient

When variables are inside absolute values, a big unknown is whether the variables are positive or negative. If x and y are both positive, then the answer to the question will be yes. But now suppose that x is 3 and y is -7. 3 - (-7) = 10. In this case, the answer to the question is no. We have a yes case and a no case. The statement is insufficient.



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FDPs

Dinner cost \$230 including a 15% tip. How much was dinner without the tip?

g

## FDPs

Percents

#### Answer: \$200

If \$230 includes the cost of the dinner plus an additional 15%, then it is 115% of the cost of the dinner, so 230 = (115/100)x.

$$\frac{100}{115} \times \frac{2}{230} = \frac{\frac{1}{115} \times \frac{1}{100}}{\frac{100}{115}} \times \frac{\frac{100}{115}}{\frac{1}{115}}$$

$$200 = x$$

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### Answer: 0.01 or 10<sup>-2</sup>

 $(10^4)(0.000001) =$ 

An easy shortcut when dealing with powers of 10 is to simply move the decimal over the same number of units as the exponent. In this case, the exponent is 4, so we move the decimal to the right 4 places. Alternatively, .000001 can be rewritten as  $10^{-6}$ , and  $(10^{4})(10^{-6}) = 10^{-2}$ .



1

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Powers of 10

## FDPs

Which number is closest to 7% of 1,440?

a) 50

b) 75

**c)** 100

# g

## FDPs

The original price of an iPhone<sup>®</sup> was increased by 25%. A sale brought the price of the iPhone<sup>®</sup> back down to its original price. The sale reduced the new price of the iPhone<sup>®</sup> by what percent?

## g



Benchmark Values

Answer: c) 100

We can save time by estimating. 1,440 is approximately 1,400, which is  $14 \times 100$ . 7% of  $(14)(100) = (7/100)(14)(100) = 7 \times 14 = 98$ . This is a slight underestimate, so answer choice c) must be correct.

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FDPs

#### Answer: 20%

Start with a smart number. Assume the price of the iPhone® is \$100. 25% of 100 is 25, so the increased price was \$125. We know the sale then reduced the price of the phone to its original price, \$100, so the sale reduced the price by \$25, because 125 - 100 = 25. The percent decrease is the difference in prices divided by the original price. 25/125 reduces to 1/5, which is 20%.



3



A bag of jellybeans contains 4 flavors: watermelon, cherry, orange and pear. 1/4 of the jellybeans are watermelon, 1/3 are cherry, 1/6 are orange, and the rest are pear. What percent of the jellybeans are pear?



## What is 35% of 120?

## FDP

**FDP** Connections

Answer: 25%

First we need to find out what fraction of the jellybeans are not pear flavored. We have to add the fractional amounts of the other flavors. The common denominator is 12, so

 $\frac{3}{12} + \frac{4}{12} + \frac{2}{12} = \frac{9}{12} = \frac{3}{4}$ . Thus, 1 - 3/4 = 1/4 of the jellybeans must be pear. 1/4 expressed as a percent is 25%.

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7



### **Benchmark** Values

#### Answer: 42

Although 35% of a number is not easy to find without some calculation, 10% and 5% are usually easier.

 $35\% = 3 \times 10\% + 5\%$ 

10% of 120 is 12 and 5% is half of 10%, so 5% of 120 is 6.

 $3 \times (12) + 6 = 36 + 6 = 42$ 



## FDPs

## What is the units digit of $(2^7)(7^4)(5^6)$ ?

# g



### Units Digit

#### Answer: 0

Although you could multiply everything out, that is too timeconsuming. Notice that  $2 \times 5 = 10$ . That means the units digit is 0. Anything multiplied by 0 is 0, so we know that the units digit of the final product will be 0.





#### Is the statement sufficient?

Last year, John earned a combined \$150,000 from his salary and bonus. This year, the amount he earned from salary was the same percentage of his total earnings as it was last year. How much was John's salary this year?

1) Last year, John earned twice as much from his salary as he did from his bonus.

## g

Percents

## FDPs

#### Answer: Insufficient

We do have enough information to determine the amount John earned from salary and from bonus **last** year. (\$100,000 comes from salary and \$50,000 from bonus.) But we are only told that the same percentage of his total earnings this year came from salary. Lacking the actual amount of salary, bonus, or total earnings, we do not have enough information to answer the question. The statement is insufficient.



## FDPs

21,267 is approximately what percent of 10<sup>6</sup>?

a) 0.2% b) 2% c) 20%

g

## FDPs

Which of the following is closest to 23% of 41/60 of 240 rounded to the nearest integer?

a) 24 b) 39 c) 52 d) 68

## g



Benchmark Values

Answer: b) 2%

Use benchmark values to estimate.  $10^6 = 1,000,000$ . Finding 1% is the same as dividing by 100, so 1% of  $10^6$  is  $10^4$  or 10,000. Since 21,267 is a little more than twice 10,000, so 21,267 is approximately 2% of  $10^6$ . You could also use heavy division to estimate your answer:

$$\frac{21,267}{10^6} = \frac{21,267}{1,000,000} \approx 2\%$$

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11



#### Answer: b) 39

The answer choices are far apart, so we can save time by estimating. 41/60 is close to 40/60, which is 2/3.  $240 \times 2/3$  = 160. 23% is close to 25%. To calculate 25% of a number, just divide by 4. 160/4 is 40. The best answer is b) 39.



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Estimation



# $\frac{(57)(10^3)(0.001)}{(10^4)(10^{-2})} =$

# g

## FDPs

Is the statement sufficient?

The combined revenue for a company for 2006 and 2007 was \$700,000. What percent of the combined revenue was earned in 2006?

1) Revenue dropped 25% from 2006 to 2007.

g

Percents

## FDPs

Powers of 10

#### Answer: 0.57

First, change 0.001 to  $10^{-3}$ . Now, combine the terms on the top and the bottom.  $(10^3)(10^{-3}) = 10^0 = 1$   $(10^4)(10^{-2}) = 10^2$ We are left with  $\frac{(57)(1)}{(10^2)}$ . To divide by  $10^2$ , just move the decimal to the left 2 places. 57 becomes 0.57.

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13



#### Answer: Sufficient

Let's label the revenue for 2006 as x and the revenue for 2007 as y. From the question, we know that x + y = 700,000. From the statement, we know that revenue dropped 25% from 2006 to 2007, which means the revenue from 2007 is only 75% of the revenue for 2006. Thus 0.75x = y. We can substitute this into the original equation to find x + (0.75x) = 700,000 and solve for x.



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FDPs

The price of a television increased from \$180 to \$216. What is the percent increase in the price?

g

Percent Change



Answer: 20%

Percent change is equal to change divided by original value. The change is 216 - 180 = \$36. The original price is \$180. 36/180 reduces to 1/5, which is the same as 20%.



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FDP

Answer: 30%

Benchmark Values

 $1.5 \times 10^7 = 15,000,000$ . We can use benchmark values to estimate. 10% of 15,000,000 is 1,500,000. This is too small. But notice that 1,500,000 is 1/3 of 4,500,000, so if we triple 10% of 15,000,000, we'll have our answer. Therefore, 4,500,000 is 30% of  $1.5 \times 10^7$ .



15

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What percent of  $1.5 \times 10^7$  is

4,500,000?



## Is the statement sufficient?

What is the units digit of  $9^x$ ?

1) *x* is a prime number.

## g

## FDPs

The price of a refrigerator is increased by 50%. It then goes on sale, with the new sale price equaling 75% of the original price. The sale price is what percent of the increased price?

g

Percents

## FDPs

Last Digit

#### Answer: Insufficient

When trying to find the units digit of a number, ignore all the other digits in a number.  $9^1 = 9$ ,  $9^2 = 81$ ,  $9^3 = 729$ ,  $9^4 = 6,561$ . The units digit of 9 raised to the first four powers is 9, 1, 9, 1, etc. We see that the pattern repeats: odd exponents yield a units digits of 9, while even exponents yield a units digit of 1. We know that x is prime. Although all other primes are odd, 2 is even. Thus we cannot determine the units digit, and the statement is insufficient.

17



#### Answer: 50%

When solving word problems involving percents, it's usually helpful to pick 100 as your starting value. If the price is increased by 50%, the new price is \$150. The sale reduces the price to 75% of the original price. \$100 is the original price, so the sale reduces the price to \$75. The question asks what percent the sale price is of the increased price. 75/150 = 1/2 = 50%.



## FDPs

## What is the units digit of $(5^3)(7^2)(3^2)$ ?

# g



Last Digit

#### Answer: 5

When solving for the units digit of a number, you can ignore all the other digits.

 $5^3 = 125$ . Drop the other digits and keep the 5.  $7^2 = 49$ . Drop the other digits and keep the 9.

$$3^2 = 9$$
. Keep the 9.

$$5 \times 9 = 45$$
. Keep the 5

 $5 \times 9 = 45.$ 

The units digit is 5.

ManhattanGMAT Fractions, Decimals & Percents Strategy Guide Digits & Decimals Copyright © 2009 MG Prep, Inc. on her total sales revenue exceeding \$50,000. How much did she make on commission this year?

1) If her total sales revenue had been 25% higher, her commission would have been 20% higher.

Carla earns a base salary of \$30,000 plus 10% commission

## FDPs

## FDP Connections

#### Answer: Sufficient.

Is the statement sufficient?

First, label total commission *c* and total sales revenue *r*. The key is to realize that we have 2 different ways to express the relationship between our two variables. From the question, we know that c = 0.1(r - 50,000). From the statement, we know that 1.2c = 0.1(1.25r - 50,000). We know that we have 2 linear equations relating our 2 variables, so we will get one unique solution.

For extra practice, what is the value of c and r?



19



g



Factoring Quadratic Equations

Answer: (x-5)(x-6) = 0

Since the last sign is positive, set up 2 parentheses with the sign of the middle term.

(x - )(x - )

Find two numbers that multiply to 30 and add to 11 and place them in the parentheses.

(x-5)(x-6)

What values for *x* solve the equation?

<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Quadratic Equations

## EIVs

For each of the following, could the answer be an integer if x is an integer greater than 1?

a)  $x^{10} + x^{-10} =$ 

b)  $x^{1/6} + x^{1/2} =$ 

g

## Fractional and Negative Exponents

Answer: a) No; b) Yes

a) No.  $x^{-10} = 1/x^{10}$ . For any x > 1, this won't be an integer.

**b) Yes**. This is equivalent to  $\sqrt[6]{x} + \sqrt{x}$ , so if x has an integer sixth root this will be an integer. For example, if x equals 64, the sixth root of x is 2, and the square root is 8.

Any number with an integer sixth root will have an integer square root. Why?



1

EIVs

<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Exponential Equations

Is it possible to solve for a single value of *x* in each of the following systems of equations?

a) 
$$2x + 3y = 8$$
  
 $2x - y = 0$   
c)  $2x - 4y = 13$   
b)  $x^2 + y - 17 = 0$   
 $y = 2x$ 

## EIVs

What is the 25<sup>th</sup> term of this sequence?

 $S_n = S_{n-1} - 10$  and  $S_3 = 0$ .

## g

## EIVs

**Basic** Equations

Answer: a) Yes; b) No; c) No

-6x + 12y = -39

a) Yes. We are given 2 linear equations. There are no xy terms or x/y terms.

**b)** No. There is an  $x^2$  term. Even if 2x is substituted into the first equation for *y*, 17 isn't a perfect square, so we should expect the quadratic to have 2 distinct solutions.

c) No. The two equations are equivalent. The second equation is just the first equation multiplied by -3.



### Linear Sequences

#### Answer: -220

First, we need to convert the recursive sequence definition provided into a direct sequence formula. Each term is 10 less than the previous one. Therefore  $S_n = -10n + k$ , where k is some constant that we must determine. Use  $S_3$  to find a value for k: 0 = -10(3) + k. Thus, k = 30, so  $S_n = -10n + 30$ . Now we plug in 25 for n:  $S_{25} = -10(25) + 30 = -220$ .

Alternatively, we could plug in 0 for  $S_3$  and find that  $S_4 = -10$ ,  $S_5 = -20$ ,  $S_6 = -30$ , etc. Thus,  $S_{25} = -220$ .



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Formulas

## Is the statement sufficient?

What are the solutions to the equation  $x^2 + kx - 10 = 0$ , where k is a constant?

(1) One of the solutions is -5.

g



Is the statement sufficient?

Is x > y?

(1) ax < ay

## g



Factoring Quadratic Equations

#### Answer: Sufficient

If one solution is -5, we know one of the factors of the quadratic expression is (x + 5). We now know the other factor is (x - 2) because the two numbers in parentheses must multiply to -10. Therefore the other solution is x = 2. The statement is sufficient.



5



Positives & Negatives

#### Answer: Insufficient

We do not know the sign of *a*, so we cannot simply divide by *a* on both sides. We must consider two possible scenarios when rephrasing statement (1). If a > 0, then we can divide by *a* on both sides and x < y. However, if a < 0, after dividing we flip the inequality sign and get x > y. The statement is insufficient.



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Inequalities

Solve for *y*:

$$y^2 + 7y - 60 = 0$$

g



Factoring Quadratic Equations

Answer: y = -12, 5

Since the last sign is negative, set up 2 parentheses with opposite signs. (y + )(y - )Find two numbers that multiply to 60 and subtract to 7:  $12 \times 5 = 60$  12 - 5 = 7Place the larger number in the parentheses with the same sign as the middle term (+7y): (y + 12)(y - 5) = 0If y + 12 = 0, then y = -12. If y - 5 = 0, then y = 5.

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7



What is the value of *x*?

 $5^{3x} = 5^{7x-4}$ 

## g

## Same Base

#### Answer: 1

EIVs

Since the bases are equal, we can simply set the exponents equal to each other.

$$3x = 7x - 4$$
$$4 = 4x$$
$$1 = x$$

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Exponential Equations

What is the minimum value of  $f(x) = -5 + (x + 7)^2$ , and at what value of x does it occur?

g



Quadratic Functions

Answer: minimum value = -7, x = -5

The squared expression will always be non-negative, so to make f(x) as small as possible, make the squared expression as small as possible – set it equal to zero. If x + 7 = 0, x = -7. Once you have the x value, plug it back into the original equation to solve for the minimum value.  $f(x) = -5 + (0)^2$ . Therefore, the minimum value is -5.

Remember, f(x) and y are synonymous.

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Functions EIVs

## What are all possible values of x?

 $x^2 - 27x + 50 = 0$ 

g



Factoring Quadratic Equations

Answer: x = 2 or 25

Since the last sign is positive, set up 2 parentheses with the sign of the middle term.

(x - ) (x - )

Find two numbers that multiply to 50 and add to 27 and place them in the parentheses. (x-2) (x-25) = 0.



9

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Quadratic Equations





## Simplify:

a) 
$$4^5 + 4^5 + 4^5 + 4^5$$

b) xw + yw + zx + zy

g

Factoring

## EIVs

Inequalities

Answer:  $b \leq -28$ 

To isolate b, multiply both sides by -7 and flip the direction of the inequality sign.

When multiplying or dividing an inequality by a negative number, remember to switch the direction of the inequality sign.

#### ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Inequalities

11

## EIVs

#### Answer: a) $4^6$ ; b) (w + z)(x + y)

a) The greatest common factor is  $4^5$ .  $4^5(1 + 1 + 1 + 1) = 4^5(4) = 4^6$ .

Make sure to look for common terms that can be factored out of an expression. Factoring is often a crucial step toward solving an equation.

b) Factor by grouping: (xw + yw) + (zx + zy) = w(x + y) + z(x + y) = (w + z)(x + y).

If you have 4 expressions and 4 variables, look to factor by grouping.



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Basic Equations



Solve for each of the following:

a) If 
$$x = \frac{7 - y}{2}$$
, What is  $2x + y$ ?

b) If  $\sqrt{2t + r} = 5$ , What is 3r + 6t?

g



Distribute:

(b + 7)(b - 10)

g

FOIL

## EIVs

MADS Manipulations

Answer: a) 7; b) 75

- a) Multiply both sides by 2 and add *y* to each side.
  2x + y = 7
- b) Square both sides and multiply by 3. 6t + 3r = 75



Answer:  $b^2 - 3b - 70$ 

Use FOIL – First, Outer, Inner, Last

(b)(b) + (b)(-10) + (7)(b) + (7)(-10)

 $b^2 - 10b + 7b - 70$  $b^2 - 3b - 70$ 

> ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Quadratic Equations

If 2 is one solution to the equation  $x^2 - 9x + c = 0$ , where c is a constant, what is the other solution?

g



Factoring Quadratic Equations

Answer: 7

Work backwards – even though we do not know the value of *c*, since 2 is one solution, we know the factored form of the quadratic is

(x-2)(x-2). We also know that the two numbers in parentheses must add to -9. Therefore the factored form is (x-2)(x-7) and the other solution is x = 7.

This problem can also be solved by plugging x = 2 into the original equation and solving for *c*.



What error has been made?

$$x^2 = 36$$
$$\sqrt{x^2} = \sqrt{36}$$

*x*=6

## Even Exponents

#### Answer:

FIVs

Remember,  $\sqrt{x^2} = |x|$ . So after we take the square root of both sides, we have |x| = 6.

This gives two possibilities: x = 6 or x = -6.

Alternatively, simply recall that there are always two possible solutions in exponential equations with an even exponent. Thus when  $x^2 = 36$ , x = 6 or -6.



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Exponential Equations

If c < 4, what is the range of possible values of d for the equation 3c = -6d?

g



Extreme Values

Answer: d > -2

We can actually replace *c* with its extreme value, which is "less than 4." The equation will read 3(less than 4) = -6d. So (less than 12) = -6d. Divide by -6, and remember to flip the sign, because we're dividing by a negative. Thus we have (greater than -2) = d.

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Inequalities

17



What are the roots of  $x^3 - x = 0$ ?

## g

Factoring

Answer: x = 0, -1, or 1

Factor the equation, since we already have 0 on one side:  $x(x^2-1) = 0$  x(x+1)(x-1) = 0x = 0, -1, or 1.

The temptation is to move x to the other side and divide both sides by x, leaving us with  $x^2 = 1$ . Avoid dividing away a variable unless you *know* it does not equal 0.



EIVs

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Basic Equations

Consider the formula 
$$H = \frac{2a^3}{b}$$

If *a* is doubled and *b* is increased by a factor of 4, by what factor is *H* increased?

g

## EIVs

What is x? (Hint: Try a method other than substitution)

x + y = 103x - 5y = 6

g



Formulas with Unspecified Amounts

#### Answer: 2

The exponent of 3 on *a* means when we double *a*, the whole formula gets multiplied by  $2^3$ , or 8. *b* has no exponent, but it is in the denominator, so quadrupling it is the equivalent of multiplying the formula by 1/4. Thus, *H* gets multiplied by 8  $\times 1/4 = 2$ .

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Formulas

19



### Solving by Combination

#### Answer: 7

One way to solve for a variable when you have two equations is to combine the equations in a way that eliminates one variable. In this case, we can multiply the first equation by 5, and then add it to the second equation, giving us:

$$5x + 5y - 50$$
  

$$3x - 5y = 6$$
  

$$8x + 0y = 56 \rightarrow x = 7$$

On the GMAT, combination is often faster than substitution.



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Basic Equations



Is the statement sufficient?

ls *xy* < 25?

(1) x and y are both less than 5.

## EIVs

**Extreme** Values

Answer: Insufficient

We cannot simply multiply x < 5 and y < 5 to get xy < 25. If x and y are both negative, xy could be greater than 25.

Example: (-10)(-4) = 40.

Could we multiply x > 5 and y > 5 to get xy > 25?

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21



EIVs

#### Answer: w = 15

We must first obtain the same base on both sides. Convert the 8 into a power of 2:

Solve for *w*:

 $2^{2w} = 8^{w-5}$ 

 $2^{2w} = (2^3)^{w-5} \qquad 2^{2w} = 2^{3w-15}$ 

Now that the bases are equal, we can set the exponents equal to each other: 2

$$w = 3w - 15 \rightarrow w = 15$$



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide **Exponential Equations**


g

## EIVs

Taking the Square Root

### Answer: x = 11 or -3

Do not multiply out  $(x - 4)^2$  if there is a perfect square on one side of the equation. Instead, take the square root of both sides, and remember to place the side of the equation containing the unknown in an absolute value. |x - 4| = 7. Our two solutions to this equation are x - 4 = 7 and x - 4 =-7. Solving these two equations gives us x = 11 and -3.

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23



The first few steps of a problem are shown. Finish the problem and answer the question: what is x?

 $\sqrt{x+3} = x-3$   $x+3 = (x-3)^2$   $x+3 = x^2 - 6x + 9$  $0 = x^2 - 7x + 6$ 

g

### EIVs

### Eliminating the Root

### Answer: x = 6 (x does NOT equal 1!)

Although this equation can be simplified and factored into (x-6)(x-1)=0, you need to be careful. When you square an equation containing a variable, you may create extraneous solutions. Potential answers need to be plugged back in to the original equation and verified. 6 is a genuine solution, 1 is not.

Try plugging 1 back into the original equation to verify that *x* cannot equal 1.



<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Exponential Equations EIVs

What is x + y + z?

$$x + y = 8$$
  
 $x + z = 11$   
 $y + z = 7$ 

g



Simplify:

 $(\sqrt{2} + 3)(\sqrt{2} - 3)(2 - \sqrt{3})(2 + \sqrt{3})$ 

g



Solving by Combination

### Answer: 13

There is often a faster method than solving for the value of each variable. In this case, we can simply add all the equations together!

x + y = 8 x + z = 11 y + z = 7 2x + 2y + 2z = 26x + y + z = 13

Remember, x + y + z is a "combo." In this type of problem there is a good chance you will not need to determine the individual values of the variables.





Answer: -7

Remember,  $(a + b)(a - b) = a^2 - b^2$ .

Therefore, our expression is equal to:  $(2-9) \times (4-3) = (-7)(1) = -7$ 



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Quadratic Equations

Special Products



### Is the statement sufficient?

A group of rabbits multiplies at a constant rate. By what factor does its population increase every day?

(1) The population grows from 200 to 5,000 in one week.

g



### Simplify:

$$\frac{a-b}{\sqrt{a}+\sqrt{b}}$$

g

Using the Conjugate



### Population Growth

#### **Answer: Sufficient**

Remember, we just need to know that we can calculate the rate of growth. They've given us the initial and final numbers of rabbits, as well as the time span. That is enough to calculate the rate of growth. For example, in 7 days, the population increases by a factor of 5,000/200 = 25. In one day it increases by a factor of 7/7

 $\sqrt{25}$ . (We do not, however, need to actually do this calculation on a Data Sufficiency question!)

27

## EIVs

### Answer: $\sqrt{a} - \sqrt{b}$

Anytime there is a square root term in the denominator that is added to or subtracted from another term we can multiply by the conjugate (the same expression, but with the sign on the  $2^{nd}$  term flipped) to simplify:

$$\frac{a-b}{\sqrt{a}+\sqrt{b}}\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right) = \frac{(a-b)(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{(a-b)(\sqrt{a}-\sqrt{b})}{(a-b)} = \sqrt{a}-\sqrt{b}$$

Alternatively, you could use the special product  $a^2 - b^2 = (a + b)(a - b)$  to solve. In this case,  $a - b = (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ , and so the term  $\sqrt{a} + \sqrt{b}$  would cancel from the top and bottom, leaving  $\sqrt{a} - \sqrt{b}$ 



<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy</u> <u>Guide</u> Ouadratic Equations

# EIVs

Are the two statements sufficient when combined?

What is x?

$$(1) \quad \frac{3x}{3y+5z} = 8$$

2) 
$$6y + 10z = 18$$

g



Combo Problems

### Answer: Sufficient

Divide the equation in (2) by 2 and get 3y + 5z = 9. Substitute 9 for the denominator of the fraction in (1). This leaves an equation with one variable, *x*.

Remember, when you see 3 variables and only 2 equations, you should not automatically assume that you cannot solve for a particular value.

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Basic Equations

29

EIVs

# Simplify: $\frac{3}{2+\sqrt{3}}$

### Using the Conjugate

### Answer: $6 - 3\sqrt{3}$

EIVs

To remove a square root from a denominator of the form  $a + \sqrt{b}$ , multiply the fraction by  $\frac{a - \sqrt{b}}{a - \sqrt{b}}$ . The form is the same whether you are dealing with numbers, variables, or a combination of the two.  $3(2-\sqrt{3})$   $3(2-\sqrt{3})$   $6-3\sqrt{3}$ 

$$\frac{5}{2+\sqrt{3}} \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = \frac{5(2-\sqrt{3})}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} = \frac{6-3\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3} = \frac{6-3\sqrt{3}}{1} = 6-3\sqrt{3}.$$

g i

<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Exponential Equations



### Is the statement sufficient?

In a sequence of terms in which each term is twenty-three times the previous term, what is the 11<sup>th</sup> term?

(1) The 19<sup>th</sup> term is 40.

g

# EIVs

How would you factor each of the following expressions?

a)  $x^5 - x^3$ 

```
b) 4^8 + 4^9 + 4^{10}
```

```
c) m^{n-2} - 3m^n + 4m^{n+1}
```

g

Factoring

# EIVs

**Exponential Sequences** 

Answer: Sufficient

We could simply work backwards from the 19<sup>th</sup> term, dividing each term by 23.  $S_{18} = 40/23$ ,  $S_{17} = 40/23^2$ , etc.

Generally, in a sequence, if you know the factor that each term is being multiplied by, 23 in this case, and if you know just one term, it is sufficient to solve for any other term in the sequence.



### Answer:

- a) The GCF is  $x^3$ , the smaller power.  $x^3(x^2 - 1) = x^3(x + 1)(x - 1)$ .
- b) The GCF is  $4^8$ .  $4^8(1 + 4^1 + 4^2) = 4^8(21)$ .
- c) The smallest power of *m* is the GCF. Here it is  $m^{n-2}$ :  $m^{n-2}(1-3m^2+4m^3)$ .



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Basic Equations

### EIVs

Solve by picking numbers:

Bottle 1, with capacity *x*, is half full of water. Bottle 2, with capacity *y*, is one sixth full. If Bottle 2 is three times the size of Bottle 1 and the contents of Bottle 1 are emptied into Bottle 2, how many liters of water, in terms of *y*, are in Bottle 2?

a) 
$$\frac{1}{2}y$$
 b)  $\frac{1}{6}y$  c)  $\frac{2}{3}y$  d)  $\frac{1}{3}y$  e)  $\frac{5}{6}y$ 

g



Picking Numbers

### Answer: d) 1/3 y

When problems involve many fractions and no specific quantities, it is best to pick numbers that are multiples of all the denominators in the problem. The least common multiple of 6 and 2 is 6. Thus, let the capacity of Bottle 1 = 6 and the capacity of Bottle 2 = 18. Bottle 1 holds 3 liters and bottle 2 holds 3 liters. Bottle 1 is dumped into Bottle 2, which then contains 6 liters. Test each answer choice with *y* 

= 18 and notice that d) is the solution, since  $\frac{1}{3}(18) = 6$ .

ManhattanGMAT Equations, Inequalities, & VIC's Strategy Guide VIC's

33



Set up an appropriate equation to describe the given scenario:

The elasticity (*e*) of a material is directly proportional to the square of its density (*d*) and inversely proportional to the cube of its mass (*m*).

g



Answer: 
$$e = \frac{kd^2}{m^3}$$

EIVs

A constant k is used in expressions of direct or inverse proportionality. e is directly proportional to  $d^2$ , which means  $e = kd^2$ . e is also inversely proportional to  $m^3$ , so  $e = k/m^3$ . Putting these two equations together,

we get 
$$e = \frac{kd^2}{m^3}$$
.

Note that k in the final equation must be the <u>product</u> of the k constants in the first two equations, but since k could be any value, we can repeat the use of k for simplicity.



<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Functions

### EIVs

### Is the statement sufficient?

Given that  $x^2 - y^2 = 20$ , what is y?

(1) x + y = 5

# g

# EIVs

Identify the error:

 $8!+2 \le x \le 8!+10$  implies that  $2 \le x \le 10$ .

# g

### EIVs

### Special Products

### Answer: Sufficient

Factor the special product. We know that (x + y)(x - y) = 20. Since (x + y) = 5, (x - y) = 4. We have two linear equations, so we know we can solve for x and y individually.

The equations are linear because there are no squared terms, no xy terms, and no x/y terms. The solutions are x = 4.5, y = 0.5.

35



### Compound Inequalities

### Answer: $2 \le x \le 10$ is incorrect

In a compound inequality, you must perform the same operation to *all 3 expressions*, not just the outside expressions. If you subtract 8! from all 3 expressions, you get  $2 \le x - 8! \le 10$ .



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Inequalities

# EIV EIV Is the statement sufficient? Factor: $\frac{x^2}{9} - 25y^2$ ls *xy* < 0? (1) xz > 0 and yz < 0

## EIVs

Positives & Negatives

### Answer: Sufficient

xz > 0 means x and z have the <u>same sign</u>. yz < 0 means y and z have <u>opposite signs</u>. Together, this means that x and y must have opposite signs and consequently xy < 0.

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Inequalities

### EIVs

### Special Products

# Answer: $\left(\frac{x}{3} + 5y\right)\left(\frac{x}{3} - 5y\right)$

This expression is a slightly more complicated version of the special product  $a^2 - b^2 = (a + b)(a - b)$ . Notice that  $x^2$ , 9, 25, and  $y^2$  are all perfect squares:

$$\frac{x^2}{9} - 25y^2 = \left(\frac{x}{3}\right)^2 - \left(5y\right)^2 = \left(\frac{x}{3} + 5y\right)\left(\frac{x}{3} - 5y\right)$$



<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Quadratic Equations EIVs

The first three terms of a linear sequence are -2, 18, and 38. What is the rule for this sequence?

g



If  $10 \le m \le 20$  and  $-2 \le p \le 15$ , and mand p are both integers, what is the maximum possible value for m - p?

# g



Linear Sequences

#### Answer:

Since the terms are increasing by 20, we know the rule is  $S_n = 20n + k$ . Use any of the three given terms to solve for k:

 $S_2 = 20(2) + k$  18 = 40 + k k = -22

The rule is  $S_n = 20n - 22$ .

Try solving for k using S<sub>1</sub> or S<sub>3</sub> and verify that you get the same value. Could you express this sequence using a recursive definition?





Extreme Values

Answer: 22

To maximize m - p, make *m* as large as possible and make *p* as small as possible. m = 20 and p = -2. 20 - (-2) = 22.



39

ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide Inequalities

### EIVs

### Is the statement sufficient?

What are the solutions to  $x^2 - 10x + b = 0$ ?

1) The sum of the roots is 10.

g

# EIVs

Solve by picking numbers and calculating a target:

What is the average of  $(x + y)^2$  and  $(x - y)^2$ ?

a)  $2x^2 - 2y^2$ b)  $x^2 + 4xy + y^2$ c)  $x^2 + y^2$ 

# g



Factoring Quadratic Equations

Answer: Insufficient

Since the middle term of the quadratic expression is -10x, we know the factored form would take the form (x - a)(x - b), where a + b = 10. Thus we already knew the sum of the roots is equal to 10 before statement (1), so it is not enough information.

<u>ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide</u> Quadratic Equations

41



### Picking Numbers

### Answer: c)

Let's pick 
$$x = 3$$
 and  $y = 2$ .  
 $(3 + 2)^2 = 25$  and  $(3 - 2)^2 = 1$ , so the average is  $\frac{25 + 1}{2} = 13$ .

Now, let's test each answer choice: a)  $2(3)^2 - 2(2)^2 = 10$ b)  $(3)^2 + 4(3)(2) + (2)^2 = 37$ c)  $(3)^2 + (2)^2 = 13$ 



ManhattanGMAT Equations, Inequalities, & VICs Strategy Guide VIC's



Everyone in a certain office orders a cup of coffee. The ratio of cappuccinos to lattes to espressos ordered is 1:2:3. If there are 60 people in the office, how many cups of each type of coffee were ordered?

g



The average of 177, 176, 189 and *x* is 180. What is *x*?

# g



### Unknown Multiplier

### Answer: 10 cappuccinos, 20 lattes, 30 espressos

Using the unknown multiplier, we can set up the equation 1x + 2x + 3x = 60. Solving for x, we find that x = 10, and then we can apply that multiple to each element in the proportion.

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### Balancing Method

### Answer: 178

A shortcut to dealing with averages is to focus on how much above or below the average every number in the set is. Then, we can "balance" this difference in the final term. In this example, 177 is 3 below the average, 176 is 4 below the average, and 189 is 9 above the average. -3 - 4 + 9 = +2, so the fourth number must be 2 *below* the average to balance it out. 178 is 2 below 180.



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Which is the correct expression?

The phone call pricing for a long distance company is as follows: \$5.00 for the first minute, and \$0.15 for every additional minute. After 10 minutes, the price drops to \$0.10 per minute. How much does a 17 minute phone call cost, in dollars?

a) 5 + 10(0.15) + 7(0.10) b) 5(10) + 0.15 + 7(0.10) c) 5 + 9(0.15) + 7(0.10)

g



In a round of miniature golf, the probability that Jasper will get a hole in one is 1/13. The probability that Cornelius will get a hole in one is 1/12. What is the probability that neither of them will get a hole in one on the next hole?

# g

# WT

Algebraic Translations

Answer: c) 5 + 9(0.15) + 7(0.10)

We know that the first minute costs 5.00. The next 9 minutes (not 10 – don't forget to subtract out the first minute!) will be charged at the rate of 0.15 per minute. After that, the next 7 minutes will be charged at the rate of 0.10 per minute.



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### Answer: 11/13

If Jasper has a 1/13 chance of getting a hole in one, that means he has a 12/13 chance of not getting it. Similarly, Cornelius has an 11/12 chance of not getting the hole in one. Because we want the probability of Jasper not getting the hole in one AND Cornelius not getting the hole in one, we

multiply the probabilities:  $\frac{12}{13} \times \frac{11}{12} = \frac{11}{13}$ .



3

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Which of the following changes to a set of at least 3 consecutive positive integers will result in a list whose median and mean are different?

- a) Every number in the list is tripled
- b) Every number in the list has 12 added to it
- c) Every number in the list is squared

g



Initially, the ratio of potbellied pigs to carrots in a room had been 1:60. After the potbellied pigs ate most of the carrots, though, the new ratio was 3:1. If there were 6 potbellied pigs in the room, how many carrots did they eat, total?

# g

Ratios



### Evenly Spaced Sets

### Answer: c) every number in the list is squared

As long as a set of numbers is evenly spaced, its average will equal its median. The changes described in answer choices a) and b) would keep the numbers in the lists equally spaced. Only answer choice c) would change the spacing.

# WT

### Answer: 358 carrots

From the first ratio, we know that there were originally 360 carrots, since 1/60 = 6/360. The second ratio tells us that the potbellied pigs only left 2 carrots uneaten, since 3/1 = 6/2. We can calculate that 360 carrots – 2 uneaten carrots = 358 carrots.



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5



# WT

Sean is 15 years older than Eric. In 6 years Sean will be twice as old as Eric. How old is Eric?

a) 9

b) 14

c) 24

# g



Is the statement sufficient?

Two sets, A and B, have the same number of elements and the same median. Which set has the higher average?

1) In Set A, 75% of the numbers are greater than or equal to the median. In Set B, 50% of the numbers are greater than or equal to the median.

# g



Using the Answer Choices

### Answer: a) 9

An alternative approach to setting up equations and solving for the appropriate variable is to use the answer choices to help you. To demonstrate using the right answer, we begin by assuming that Eric is 9. From the first sentence, we know that Sean is 24. In 6 years, Eric will be 15 and Sean will be 30. 30 is indeed 2 times 15, so we know that 9 is the right answer.

For this type of question, using the answer choices is an alternate approach to setting up the equations. To do very well on the GMAT, it's important to know how to setup and solve the algebra, but you should also have other tools available to you.



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7



### Median and Mean

### Answer: Insufficient

Although both sets have the same number of elements, it is important to remember that the median only tells you the middle number in the list. It tells you nothing about the distribution of the other numbers in the list.

For added practice, create two pairs of sets A and B. In one set, have the average of A be higher than the average of B. In the other, have the average of B be higher than the average of A.



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Jeff, completed a 40 mile circuit around the city. He jogged at a constant speed of 5 mph, Then Jeff completed the same circuit again, but this time he biked at a constant speed of 20 mph. What was Jeff's average speed for the total trip?

g

Average Speed



#### Answer: 8 mph

To calculate average speed, you need the total distance traveled and the total time spent traveling. The distance is straightforward. Jeff traveled 40 miles twice, for a total distance of 80 miles. If we divide distance by speed, we find that it took him 40/5 = 8 hours to jog the circuit, while it took him 40/20 = 2 hours on the bike, for a total time of 10 hours.

Finally, the average speed is the total distance divided by the total time:  $\frac{80 \text{miles}}{10 \text{hours}} = 8 \text{ mph.}$ 

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9



Is the statement sufficient?

Eight participants in a psychology experiment are given different amounts of money at the beginning of the experiment. Will the average amount of money each participant has at the end of the experiment be higher than, lower than, or equal to the average that each participant had at the beginning of the experiment?

1) Half of the participants are asked to give 20% of the original money back, while the other half are given an additional 25%.

# g

Mean

# WT

### Answer: Insufficient

It's important to remember that the average does not give us <u>any</u> information about any individual number in the set. We only know the percent by which all the values were changed, but we don't know the distribution of the money among the participants. Without that information, we have no way of knowing what effect those changes will have on the average after the experiment. The statement is insufficient.

For added practice, assign sets of payouts to each of the participants that result in the average amount increasing. Then, assign payouts that result in the average amount decreasing.



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Greg and Liz are currently 350 miles apart. They begin driving toward each other, Greg driving 60 mph and Liz driving 40 mph. How long will it take until they meet?

g



Which is the correct expression for this problem?

8 students have been chosen to play for PCU's inter-collegiate basketball team. If every person on the team has an equal chance of starting, what is the probability that both Tom and Alex will start? (Assume 5 starting positions)



WT

### Combined Rates

#### Answer: 3.5 hours

When dealing with two objects moving toward each other, we can simplify the calculations by adding the rates together. If Greg is moving 60 mph and Liz is moving 40 mph, then together they are traveling 100 miles per hour relative to one another. At that speed, they will collectively travel 350 miles in 3.5 hours, because T = (350 miles)/(100 mph) = 3.5.

11

### Combinatorics/Probability

# Answer: a) $\frac{\frac{6!}{3!3!}}{\frac{8!}{5!3!}}$

There are two tasks in this problem: determine how many different combinations there are of all the players (denominator) and determine how many of those combinations include two specific players (numerator). For the total number of combinations, there are 8 people to choose from, and each person is

either starting or not starting. Thus there is one group of 5 and one group of 3. To calculate how many combinations include both Tom and Alex, we assume that they are on the team, and that means we are now choosing among 6 people for the remaining 3 spots on the team, while there are still 3 people not on the team.



ManhattanGMAT Word Translations Strategy Guide Probability



5 people in a company earn an average salary of \$50,000. If 2 of the employees earn an average of \$65,000, what is the average salary of the remaining 3 employees?

Solve using the "balancing" method

g



10 years ago Tina was half as old as Ron will be 7 years from now. Which equation correctly represents this relationship?

a) (1/2)(T - 10) = (R + 7)b) (T - 10) = (1/2)(R + 7)

g



### Balancing Method

#### Answer: \$40,000

One approach to this problem is to balance the "overs" and the "unders." The 2 employees making \$65,000 each make \$15,000 more than the mean, for a total of \$30,000 over the mean. That means the remaining 3 employees need to make a combined \$30,000 under the mean. Distribute that amount evenly, and the remaining salaries average out to \$10,000 below the mean, or \$40,000.

13

### WT

### Algebraic Translations

Answer: b) (T - 10) = (1/2)(R + 7)

When translating sentences into equations, one of the easiest mistakes to make is to put the multiplier in the wrong place. Take the time to verify that it is where it should be. In this case, if Tina's age 10 years ago was half what Ron's age will be in 7 years, we need to multiply Ron's age by 1/2 to make them equal.

Don't forget to use the parentheses!



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Jerry and Ross decide to have a footrace. They run 1,000 meters. Jerry runs 5 meters per second, and Ross runs 4 meters per second. Halfway through the race, Jerry realizes he is ahead and stops running for one full minute before finishing the race at the same speed. Who wins the race?

g



What is the average of 3,456, 3,463, 3,470, 3,477, and 3,484?

# g



Rates & Work

#### Answer: Ross wins

First, calculate how long it will take Ross to finish the race. T = D/R. T = (1,000 m)/(4m/s) = 250 s. To make the calculations simpler for Jerry, add 60 seconds to the total time to take into account the minute he spent not running. T = (1,000 m)/(5 m/s) + 60 s = 260 s. Ross finishes the race in less time.

15



### Evenly Spaced Sets

#### Answer: 3,470

Any time a list is composed of evenly spaced numbers, the average will equal the median. In this case, all the numbers in the list are spaced 7 units apart. The median is 3,470, and the average is also 3,470.



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### Is the statement sufficient?

6 girls are added to a class, bringing the ratio of girls to boys to 1:1. How many students are in the class?

1) The original ratio of girls to boys was 2:3.

g

# WT

Car A travels east at 60 mph. Car B is 45 miles behind Car A and also travels east at 75 mph. How long will it take Car B to catch up with Car A?

# g



### Unknown Multiplier

### **Answer: Sufficient**

Begin with the unknown multiplier. If the original ratio of boys to girls is 2:3, then the original number of girls is 2x and the original number of boys is 3x. After 6 girls are added, the new ratio is 1:1, so we can write the equation  $\frac{2x+6}{3x} = \frac{1}{1}$ . Thus x = 6, and we can use that value to determine how many students are in the class. The statement is sufficient.

17



### Combined Rates

#### Answer: 3 hours

When solving a combined rates problem with 2 objects moving in the same direction, we can ignore their actual speeds and focus only on the difference between their speeds. Car B is going 15 mph faster than Car A. That means it will catch up to Car A at a rate of 15 mph. Car B is currently 45 miles behind Car A, and  $R \times T = D$ , so (15 mph)  $\times T = 45$  miles. Solving for T gives us T = 3 hours.



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A shipping company charges  $5 + 10/y^2$  dollars per package shipped by a customer over a given month, where y is the number of packages shipped that month by the customer. If a customer spends \$51 one month on shipping, which equation will correctly solve for the number of packages shipped that month by the customer?

a) 5y + 10 = 51 b) 5 + 10/y = 51 c) 5y + 10/y = 51

g

# WT

Jeff can build a doghouse in 6 hours. Kevin can build the same doghouse in 3 hours. How long will it take them, working together, to build 1 doghouse?

# g

## WT

### Algebraic Translations

Answer: c) 5y + 10/y = 51

We know that *y* represents the number of packages shipped by the customer in the month, and  $5 + 10/y^2$  is the shipping cost per package for that customer. Therefore, the product of these two terms must equal 51:

 $(y)(5 + 10/y^2) = 51$ 5y + 10/y = 51

For added practice, what is the value of y? Try choosing integers that make sense.

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19



### Combined Work

### Answer: 2 hours

Whenever you are told how long it takes a person to complete a task, a great first step is to turn that information into a rate. In an  $R \times T = W$  equation, we can think of completing the task as doing 1 unit of work. If it takes Jeff 6 hours to build the doghouse, then his rate is 1/6 of the doghouse per hour. Similarly, Kevin's rate of work is 1/3 of the doghouse per hour. When we combine their rates (because they're working together), we see that they complete 1/2 of the doghouse every hour, because 1/6 + 1/3 = 1/2. At that rate, they will complete the doghouse in 2 hours.



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In the World's Strongest Man competition, Olav Gundersson managed to pull a truck a total distance of 85 ft combined in two tries. On his second try, he pulled the truck 10 ft more than half the distance he pulled the truck on his first try. How far did he pull the truck on his first try?

a) 45 ft b) 50 ft c) 55 ft d) 60 ft e) 65 ft

g



Is the statement sufficient?

John and Fawn, each drinking at a constant pace, can finish a case of soda together in 12 hours. How fast can John, drinking alone, finish the case?

1) It would take John 10 more hours to finish the case drinking alone than it would for Fawn to finish the case.

# g



Using the Answer Choices

### Answer: b) 50 ft

We can set this problem up and solve algebraically. We know that the sum of the two tries equals 85 feet, and the second try is 10 more than half the first try. Let's use x to represent the first try and y to represent the second try:

x + y = 85 y = 10 + 0.5x x + (10 + 0.5x) = 85 1.5x = 75x = 50

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21



### Combined Rates

### Answer: Sufficient

The key to this problem is to realize that we have 3 variables, and 3 equations describing their relationships. The 3 variables are **1**) the rate at which John drinks,  $(r_1)$  **2**) the rate at which Fawn drinks,  $(r_2)$  and **3**) the time it takes John to drink the case of soda (*t*). The 3 equations are: **1**)  $r_1 + r_2 = 1/12$  **2**)  $r_1 \times t = 1$ **3**)  $r_2 \times (t-10) = 1$ 

With 3 equations and 3 variables, we will be able to solve for unique values for each variable. The statement is sufficient.



ManhattanGMAT Word Translations Strategy Guide Rates & Work



### Is the statement sufficient?

A bag contains ping pong balls, each with a number written on it. The average of all the numbers is 50. Some of the ping pong balls are removed. What is the average of the numbers on the balls still remaining in the bag?

1) One third of the ping pong balls are removed from the bag, and the average of the numbers written on those balls is 20.

g

# WT

At a farmer's market, one stall is selling mangos, kumquats and rutabagas. The ratio of mangos to kumquats is 5:4, and the ratio of kumquats to rutabagas is 3:7. Which of the following could be the number of kumquats?

a) 28 b) 36 c) 42 d) 49

g

Ratios



### Weighted Averages

### **Answer: Sufficient**

Although we don't know how many ping pong balls are in the bag, we can modify the weighted averages formula to help us answer this question. The weighted average equals the sum of the weights times the data points, divided by the sum of the weights. Thus,

 $\frac{\frac{1}{3}}{\frac{1}{20} + \left(\frac{2}{3}\right)x} = 50.$  We can solve for x, which will equal the

average of the remaining numbers. The statement is sufficient.

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23



### Answer: b) 36

A common hidden constraint of word problems involving ratios is that the actual number of items must be an integer. For instance, it would not make sense in this context to have 5 1/3 mangos. The correct answer will be a multiple of both 4 and 3, based on the ratios involving kumquats.

For added practice, if there are 36 kumquats, how many mangos and rutabagas are there?



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### Is the following statement sufficient?

In 2006, John paid 4% state taxes on his gross income. How much state tax did he pay in 2007?

1) The state tax increased to 5% of income in 2007, and his gross income increased 20% in 2007.

g

## WT

Is the statement sufficient?

Train A and Train B are 500 miles apart and are traveling towards each other at constant speeds. They meet in 5 hours. How fast is Train A going?

1) If Train B were going twice as fast, the two trains would meet in 4 hours.

# g



### Algebraic Translations

### Answer: Insufficient

For these types of word problems, it is important to translate the information into variables and try to write equations about the variables/unknowns. In this problem, we can proceed as follows:

 $TaxRate_{2006} = 4\%$ , and the rephrased question is: What is  $TaxRate_{2007} \times Income_{2007}$ ?

The statement tells us that  $TaxRate_{2007} = 5\%$  and that  $Income_{2007} = 1.2 \times Income_{2006}$ , but we do not know the value of  $Income_{2006}$ .



25

# WT

### Combined Rates

### Answer: Sufficient

First, label the speed of Train A as  $R_A$  and the speed of Train B as  $R_B$ . When dealing with objects moving towards each other, we can add their rates, so  $(R_A + R_B) \times (5 \text{ hrs}) = 500 \text{ miles}$ . From the statement, we

also know that  $(R_A + 2 R_B) \times (4 \text{ hrs}) = 500 \text{ miles}$ . We have 2 equations and 2 variables, so we can solve for the speed of Train A (75 mph). The statement is sufficient.









Special Triangles

Answer:  $12 + 4\sqrt{3}$ 

Because  $\angle ABD = \angle DAB$ , side BD has length 8. Because  $\angle BDA$  is 120°,  $\angle BDC$  is 60°, so triangle BCD is a 30-60-90 triangle.

Side	Ratio	Length
CD	$\mathcal{X}$	4
BC	$x\sqrt{3}$	$4\sqrt{3}$
BD	2x	8

C	r
ç	

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1

### GEO

### Area (Triangles)

### Answer: Sufficient

Base and height of a triangle *must* be perpendicular to each other. If PQR were a right triangle, side QR would be the base and side PQ would be the height, and the area would be 1/2(5)(12) = 30. Thus, triangle PQR is a right triangle, and the statement is sufficient.

If *PQR* were anything other than a right triangle, the height of the triangle would be less than 12, and the area would be less than 30.



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# What is the area of a circle that has a circumference of $14\pi$ ?

g



Circles

Answer:  $49\pi$ 

The key to this problem is to find the radius, because the radius is used to calculate both circumference and area.

 $C = 2\pi r$ , so  $14\pi = 2\pi r$ . Therefore r = 7.  $A = \pi r^2$ , so  $A = \pi (7)^2 = 49\pi$ 



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First, complete the ratio. 5/4 = 10/PQ. Side PQ has length 8. With two sides of a right triangle, you can use the Pythagorean theorem to find the third side. In this case, however, you should recognize that this triangle is a common Pythagorean triplet—a 6-8-10 triangle. Side QR has length 6.



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A cylinder has a volume of 10π. If the radius is doubled and the height remains the same, what is the volume of the new cylinder?

g



Volume (Cylinders)

#### Answer: $40\pi$

 $V=\pi r^2 h$ , so  $10\pi=\pi r^2 h$ . We are told the radius is doubled, so the new radius is 2r. Replace r with 2r and the new equation is  $V=\pi(2r)^2 h$ . The new volume is  $V=4 \pi r^2 h$ . Since we know that  $\pi r^2 h=10\pi$ , we know that  $V=4(10\pi)=40\pi$ . Alternately, try picking numbers for the cylinder that would produce an initial volume of  $10\pi$ ; a height of 10 and a radius of 1 are simplest. Double the radius and calculate the new volume.

5



What is the surface area of a cube with side length 5?

# g



### Answer: 150

GEC

Surface area is the sum of all the areas of the faces of the cube. The faces of a cube are all identical, so the surface area is the area of one of the faces times 6, because a cube has 6 faces. The area of one face is  $A = \text{length} \times \text{width}$ .

Therefore, surface area =  $6 \times 5 \times 5 = 150$ .



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Note: Figure not drawn to scale

Is the statement sufficient?

What is the area of the circle?

1)  $\angle$  PQR is a right angle

g



### **Right Triangles**

#### Answer: Sufficient

Triangle OQR is a right triangle, and we have 2 sides, so either by using the Pythagorean Theorem or by recognizing it as a 30-60-90 triangle, we can find the length of OQ, which is also the height of triangle PQR. If PQ = QR, then  $\angle OPQ = \angle ORQ$ . Additionally, we know that  $\angle POQ$  $= \angle ROQ = 90^\circ$ . Therefore  $\angle OQP = \angle OQR$ , because when two triangles have two angles in common, they must have common third angles. Triangles OQR and OQP are thus similar triangles, and also have identical side lengths. OP must equal 3, so we have the length of the base as well as the height. We can calculate the area of triangle PQR, so the statement is sufficient.

7



### Answer: Sufficient

If  $\angle$ PQR is a right angle, then segment PR must be a diameter of the circle. We can solve for the length of PR using the Pythagorean Theorem, or by recognizing that triangle PQR is a 3-4-5 triangle. If the diameter of the circle is 5, then the radius is 2.5. We can solve for the area of the circle using the formula  $A = \pi r^2$ , so the statement is sufficient.



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O is the center of the circle. The area of the circle is  $81\pi$ . What is the length of line segment PR?

g



Special Triangles

 $\mathbf{O}$ 

Answer:  $9\sqrt{2}$ 

If the area of the circle is  $81\pi$ , then  $81\pi = \pi r^2$ , so r = 9. OP and OR are both radii, and so both have length 9. If triangle OPR is a right triangle and has 2 sides with equal length, then it is a 45-45-90 triangle, and the ratio of the sides to the hypotenuse is  $1:\sqrt{2}$ . Therefore, the length of PR is  $9\sqrt{2}$ .

9





### Parallel Lines

Answer: Insufficient

Don't trust the picture! Although lines  $\ell$  and m appear to be parallel, nothing in the question tells us that they are. Without knowing whether the lines are parallel, we have no way to determine the value of y.



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11

Polygons

### GEO

An empty cylindrical swimming pool has a height of 10 ft; its base has an area of 15 ft<sup>2</sup>. If water fills the pool at a rate of 25 ft<sup>3</sup> every 10 minutes, how long will it take for the pool to be filled?

g



What is the length of the main diagonal of a rectangular solid with sides of length 4, 4 and 2?

# g



Volume (Cylinders)

Answer: 60 Minutes

 $V = \pi r^2 h$ . Therefore,  $V = (15 \text{ ft}^2)(10 \text{ ft}) = 150 \text{ ft}^3$ . The pool fills at a rate of 25 ft<sup>3</sup> every 10 minutes, and since  $25 \times 6 = 150$ , we can proportionally apply this to the rate:  $10 \times 6 = 60$  minutes.

13



### "Deluxe" Pythagorean Theorem

#### Answer: 6

We could use the Pythagorean Theorem twice, or we can save some time by using the "Deluxe" Pythagorean Theorem to find the interior diagonal of a rectangular prism (a box):  $d^2 = x^2 + y^2 + z^2$ .

So 
$$d^2 = 4^2 + 4^2 + 2^2 = 16 + 16 + 4 = 36$$
, and  $d = 6$ .



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The diagonal of a rectangular flowerbed is  $15\sqrt{2}$  feet. What is the area of the flowerbed, if its length and width are equal?

# g



Slope of a Line

### Answer: Sufficient

Although we cannot easily see the point (15, 6) on the grid given in this problem, we know that two points define a line. Because line  $\ell$  goes through point (0, 3) and point (15, 6), we can calculate the slope.

As an exercise, calculate the slope of line  $\ell$ .

g

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15



# Special Triangles

### Answer: 225

If the length and width are equal, then the flowerbed is a square, and the diagonal is the hypotenuse of a 45-45-90 triangle. The ratio of the sides to the hypotenuse is  $1:\sqrt{2}$ . If the hypotenuse is  $15\sqrt{2}$ , then the sides each have length 15, so the area is  $\ell \times w = (15) \times (15) = 225$ .



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Sectors

Answer: 60°

To determine  $\angle POR$ , we first need to determine what fraction of the circumference minor arc PR is. Circumference is  $C = 2\pi r$ , so  $C = 2\pi(3) = 6\pi$ .  $\pi/6\pi = 1/6$ , so minor arc PR is 1/6 of the circumference. That means  $\angle POR$  is 1/6 of  $360^{\circ}$ . /POR is  $60^{\circ}$ .

17



### Inscribed Angles

### Answer: 30°

First, we can calculate  $\angle OPQ$ : since OP and OQ are the radii of the circle, they must have equal length, which means the opposite angles in triangle PQO must be equal. Therefore  $\angle OPQ = \angle PQO = 60^\circ$ . Since PR is a diameter of the circle, triangle PQR must be a right triangle. Therefore,  $\angle$  PRQ equals 60°, so angle  $\angle PRQ = 180^\circ - 90^\circ - 60^\circ = 30^\circ$ .



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One way to think of the distance formula is that it's the square root of the Pythagorean Theorem. The hypotenuse is the distance, the difference in the *x*-coordinates is the horizontal leg of the triangle, and the difference in the *y*-coordinates is the vertical leg. In this case, the two legs of the triangle have lengths 4 and 3, respectively. This is a 3-4-5 triangle in disguise. The hypotenuse of the triangle is 5.

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19

Although we can use the Pythagorean theorem, the numbers are large, and the calculation will be time-consuming. Remember that multiples of Pythagorean triplets are <u>also</u> Pythagorean triplets. This is a 3-4-5 triangle in disguise. Every value has been multiplied by 9.  $3 \times 9 = 27$  and  $5 \times 9$ = 45. The value we're missing is  $4 \times 9$ , which equals 36.



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## GEO

Which of these lines is perpendicular to the line whose equation is  $y = \frac{2}{3}x + 6$ ?

a) 
$$y = \frac{3}{2}x - 6$$
  
b)  $y = \frac{-2}{3}x + 2$   
c)  $y = \frac{-3}{2}x + 7$ 

g



Is the statement sufficient?

Is point O the center of the circle?

1) ∠ POR = 58°



Note: Figure not drawn to scale

Inscribed Angles

g

### GEO

### Perpendicular Lines

Answer: c)  $y = \frac{-3}{2}x + 7$ 

When determining what lines are perpendicular, there is only one important piece of information – the slope. The slopes of perpendicular lines are negative reciprocals. The slope of the original line is 2/3, so the slope of a perpendicular line will be -3/2. The slope of the line in answer choice c) is -3/2.

21



### Answer: Sufficient

 $\angle$ PSR is an inscribed angle. If O were the center of the circle,  $\angle$ POR would be a central angle and angle  $\angle$ POR would be twice  $\angle$ PSR. Because 58° is not twice 30°, we know that point O cannot be the center of the circle. The statement is sufficient.



ManhattanGMAT Geometry Strategy Guide Circles and Cylinders Copyright © 2009 MG Prep, Inc.



Two sides of a triangle have lengths 3 and 9. Which of the following could be the length of the third side?

a) 6 b) 8 c) 13

g



Third Side Rule

#### Answer: b) 8

In any given triangle, the lengths of any two sides must add up to greater than the third side. Choice a) fails, because 3 + 6 is not greater than 9. Choice c) fails because 3 + 9 is not greater than 13. Similarly you could apply the rule that in any given triangle, the length of any side must be greater than the difference of the other two.







Area (Triangles)

#### Answer: 36

We know that AC has a length of 12, because it is equal to AB. We know that DC has a length of 6, because BD bisects AC.  $A = \frac{1}{2}$  (base) × (height), so  $A = \frac{1}{2}$  (6) × (12) = 36.



ManhattanGMAT Geometry Strategy Guide Triangles and Diagonals Copyright © 2009 MG Prep, Inc.



The circumference of the smaller circle is  $6\pi$ and the circumference of the larger circle is  $16\pi$ . What is the area of the shaded region?

g



### Area (Circles)

#### Answer: $55\pi$

To find the area of the shaded region, we need to find the area of the entire figure and subtract the area of the smaller circle. The area of the entire figure is the area of the larger circle. If the larger circle has a circumference of  $16\pi$ , then its radius is 8, because  $C = 2\pi r$ . The formula for area is  $A = \pi r^2$ , so  $A = \pi (8)^2 = 64\pi$ . Doing the same calculations for the smaller circle, we find it has a radius of 3 and an area of  $9\pi$ .  $64\pi - 9\pi = 55\pi$ .

25



Is the statement sufficient?

O is the center of the circle. PR is a diameter of the circle. What is the length of line segment QR?

1) The length of minor arc PQ is  $4\pi$ 

### Sectors

R

0

P

### Answer: Sufficient

GEO

Note that since PR is the diameter, triangle PQR must be a right triangle. Since  $\angle$ POQ is also a right angle, and 90° is 1/4 of 360°, we know that minor arc PQ is 1/4 of the circumference. The circumference of the circle is  $4 \times 4\pi = 16\pi$ .  $C = 2\pi r$ , so  $16\pi = 2\pi r$ . Thus r = 8. Note that in triangle QOR, both OQ and OR are radii, so they both have length 8. A right triangle with 2 equal sides is a 45-45-90 triangle, and so we know that the length of segment QR is  $8\sqrt{2}$ . The statement is sufficient.



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#### Area (Trapezoids)

#### Answer: Sufficient

The area of a trapezoid is  $\left(\frac{b_1 + b_2}{2}\right) \times$  (height), where  $b_1$  and  $b_2$ 

are the parallel bases of the trapezoid. We can determine the height, because a square that has an area of 16 must have sides of length 4, and BE is the height of the trapezoid. From the statement, we know the average of the lengths of the parallel bases

is  $\frac{10+4}{2} = 7$ , so the area is  $A = 7 \times 4 = 28$ . The statement is sufficient.

27



#### Area (Parallelograms)

#### Answer: 24

The area of a parallelogram is  $A = base \times height$ . If BC has length 8, then AD also has length 8. If AD is the base, then BE can be the height, because base and height must be perpendicular. We are told BE bisects AD, so AE must have length 4. We see then that triangle ABE is a 3-4-5 triangle in disguise. BE has length 3, and the area is (8)(3) = 24.



1 error

#### Identify the error:

Designed to accommodate only 20 million tons of trash, the mayor proposed that the landfill, overflowing with more than 35 million tons of trash, be expanded.

g



#### Modifiers

This question is about modifiers. Notice that "Designed to accommodate only 20 million tons of trash" does not have a subject. Therefore, whatever it is that is "designed to accommodate only 20 million tons of trash" should come right after the comma. As this sentence is written now, that's the mayor—this doesn't make sense. (Note that the subjunctive is used correctly in this sentence: the mayor proposed that the landfill <u>be</u> expanded).

**One correction:** The mayor proposed that the landfill, which was designed to accommodate only 20 million tons of trash but is now overflowing with more than 35 million tons, be expanded.

l



#### Identify the errors:

Having considered the new District Attorney as the city's "white knight," citizens of Truckton were dismayed by the ever-increasing numbers of serious crimes.

## g

2 errors

### Idioms

This question is about idioms. "Considered as" is incorrect. "Number of" is preferred to "numbers of."

**One Correction:** Having considered the new District Attorney the city's "white knight," citizens of Truckton were dismayed by the ever-increasing number of serious crimes.



SC

ManhattanGMAT Sentence Correction Strategy Guide Idioms

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1 error

#### Identify the error:

The development of a new drug for the treatment of a variety of nervous system disorders indicate that the university's research funding is producing real results.

g



Identify the errors:

I was at the circus for only half an hour when the assailant struck; ever since the stabbing, I am terribly afraid of clowns.

## g



#### Subject Verb Agreement

This question is about subject-verb agreement. The verb "indicate" disagrees with the singular subject "development." Note that this subject is separated from its verb by *four* consecutive prepositional phrases. This is a good example of why it is very important to match up every subject-verb pair rather than to play it by ear!

**Correct:** The development of a new drug for the treatment of a variety of nervous system disorders <u>indicates</u> that the university's research funding is producing real results.

3



#### Verb Tense

This question is about verb tense. In the part of the sentence before the semicolon, two things happen in the past, in a specific timeline: I was at the circus, and the assailant struck. Being at the circus is in "the past of the past," necessitating the past perfect. "I am terribly afraid" does not correctly express that the fear began in the past and continues into the present; the present perfect is called for here.

**Correct:** I <u>had been</u> at the circus for only half an hour when the assailant struck; since the stabbing, I <u>have been</u> terribly afraid of clowns.



ManhattanGMAT Sentence Correction Strategy Guide Verb Tense Copyright © 2009 MG Prep, Inc.

2 errors

Identify the errors:

He donated a kidney so his wife could live; for making the incision, the doctors shaved his stomach.

g



#### Idioms

This question is about idioms. The first sentence is missing the word "that" (the idiom "so that" indicates purpose). "For making" is incorrect when "in order to make" is the intended meaning.

**One correction:** He donated a kidney so <u>that</u> his wife could live; <u>to make</u> the incision, the doctors shaved his stomach.



5



#### Identify the error:

I sent a card thanking my eighth-grade English teacher for teaching me about grammar and inspiration to write.

## g

1 error

### SC

#### Parallelism

This question is about parallelism. Elements in a list must be parallel. "Teaching" and "inspiration" are not parallel. The lack of parallelism in this sentence not only sounds bad, it also creates an unintended meaning that the teacher taught the speaker about his or her inspiration, an unlikely occurrence.

**Correct:** I sent a card thanking my eighth-grade English teacher for <u>teaching</u> me about grammar and <u>inspiring me</u> to write.



ManhattanGMAT Sentence Correction Strategy Guide Parallelism

1 error

Identify the error:

The term "red panda" is misleading; it is not a true panda, but rather an unrelated rodent.

g



#### Pronouns

The pronoun "it" certainly matches "the term" in number. Unfortunately, however, "it" is meant to refer to the *actual* red panda, not to *the term* "red panda." The sentence is illogical.

**Correct:** The term "red panda" is misleading; <u>the red panda</u> is not a true panda, but rather an unrelated rodent.

Remember the "Deadly Five" pronouns: it, its, they, them, and their. Each time you see one of these pronouns on the GMAT, verify that the pronoun has an antecedent that appears in the sentence, matches in number, and is unambiguous and logical.

7



#### Identify the errors:

My husband, along with my sons, regret to inform you that neither of my sons are planning to attend your son's bar mitzvah.

## g

2 errors

### C

#### Subject Verb Agreement

This question is about subject-verb agreement. The verb "regret" does not match the subject "My husband." The phrase "along with my sons" is not part of this subject. (The fact that this phrase is in commas is also a clue that it is not part of the main subject. A good rule of thumb is "There are no substitutes for AND in making a compound plural subject.") Also, the verb "are" does not match the singular subject "neither" (note that "of my sons" is a mere prepositional phrase).

**Correct:** My husband, along with my sons, <u>regrets</u> to inform you that neither of my sons <u>is</u> planning to attend your son's bar mitzvah.



ManhattanGMAT Sentence Correction Strategy Guide Subject Verb Agreement Copyright © 2009 MG Prep, Inc.



1 error

Identify the errors:

Of my two brothers, Salvatore is the most intelligent. Of my three sisters, Julia is the cleverest.

g



Identify the error:

Wearing the latest in streamlined, high-tech swimwear, eight records were broken in speed swimming.

## g



#### Quantity Expressions/Idioms

"More" is for comparing two things; "most" is for three or more.

**Correct:** Between my two brothers, Salvatore is the <u>more</u> intelligent. Of my three sisters, Julia is the cleverest.



#### Modifiers/Verb Voice

This question is about modifiers. Who was "wearing the latest in streamlined, high-tech swimwear"? These people, presumably swimmers, are never mentioned. This creates a dangling modifier, and also implies nonsensically that the records themselves were wearing high-tech swimwear. Make sure that your modifier modifies something that is actually mentioned in the sentence (in noun form).

Additionally, the use of the passive voice ("were broken") is inferior to the active voice.

**Correct:** Wearing the latest in streamlined, high-tech swimwear, <u>the team</u> <u>broke eight records</u> in speed swimming.



ManhattanGMAT Sentence Correction Strategy Guide Quantity Expressions/Idioms Copyright © 2009 MG Prep, Inc.

1 error

Identify the error:

The murder was an expressing of longstanding tensions.

g



#### Parallelism

This question is about parallelism. Concrete nouns (murder) and simple gerunds (expressing) are not as parallel as they could be.

**Correct:** The murder was an <u>expression</u> of longstanding tensions.

Note that if the verb "to express" did not have a noun form ("expression") we could use a complex gerund, such as "the expressing of…" But, because the word "expression" exists, it is preferred to a complex gerund.

11



Identify the errors:

If my mother did not attend the London School of Economics, she might not meet my father, and I might not have been here now.

## g

3 errors

#### Verb Tense

This question is about verb tense, specifically within "If...Then" constructions. The speaker's mother's nonattendance at the London School of Economics is a past hypothetical; it needs to occur <u>before</u> the past tense in the "then" clause and thus must be in past perfect tense. The present tense is inadequate in "might not meet"; certainly the speaker's parents know one another, making the meeting an action that started in the past and continues into the present time. This requires a present perfect construction. "Have been here now" is complex for no good reason; the simple present tense is correct.

**One correction:** If my mother <u>had not attended</u> the London School of Economics, she might not <u>have met</u> my father, and I might not <u>be</u> here now.



<u>ManhattanGMAT Answer Correction Strategy Guide</u> Verb Tense

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#### 1 error

#### Identify the error:

After the president of the college publicly lamented the sharp drop in the value of its endowment, the head of faculty suggested that they initiate a fundraising campaign immediately.

g



#### Identify the errors:

Being the world's top collector of empty vitamin bottles, Mrs. Simpson has ten times as many bottles in her robust and famous collection than does Mr. Flanders, a mere hobbyist.

## g



#### Pronouns

This question is about pronouns. The first pronoun "its" is a possessive pronoun that refers back to the college. However, the "they" in the second sentence lacks an antecedent. There is no plural subject that appears earlier in the sentence. If we replace "they" with "it", it is unclear whether the pronoun "it" refers to the college or to the faculty, which is a collective noun.

**Correct:** After the president of the college publicly lamented the sharp drop in the value of its endowment, the head of faculty suggested that <u>the college</u> initiate a fundraising campaign immediately.

Remember the "Deadly Five" pronouns: it, its, they, them, and their.

ManhattanGMAT Sentence Correction Strategy Guide Pronouns

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13



#### Idioms

This question is about idioms. "Being" is unnecessary and should be eliminated. The second idiom being tested is "as ... as"; the current sentence incorrectly uses "as ... than."

**Correct:** <u>The</u> world's top collector of empty vitamin bottles, Mrs. Simpson has ten times <u>as</u> many bottles in her robust and famous collection <u>as</u> does Mr. Flanders, a mere hobbyist.



ManhattanGMAT Sentence Correction Strategy Guide Idioms

1 error

Identify the error:

Ritika was late to pick up her cat from the vet after abandoning her broken-down car on the side of the road, a 1989 Peugeot.

g



#### Modifiers

This question is about modifiers. The misplaced modifier "a 1989 Peugeot" should be modifying the car, but is actually modifying the road.

**One correction:** Ritika was late to pick up her cat from the vet after abandoning her broken-down car, <u>a 1989 Peugeot</u>, on the side of the road. (Alternately, "abandoning her broken-down 1989 Peugeot" would be even more concise).



Identify the error:

Calvin Coolidge, as well as Herbert Hoover, are good examples of presidents who held to the principles of laissez-faire economics.

## g

### SC

#### Subject Verb Agreement

This question is about subject-verb agreement. The subject of this sentence is "Calvin Coolidge." The phrase "as well as Herbert Hoover" is not part of the subject. The fact that this phrase is in commas is also a clue that it is not part of the main subject. A good rule of thumb here is "There are no substitutes for AND in making a compound plural subject."

**Correct:** Calvin Coolidge, as well as Herbert Hoover, <u>is a good example</u> <u>of a president</u> who held to the principles of laissez-faire economics.

**OR:** <u>Calvin Coolidge and Herbert Hoover are</u> good examples of presidents who held to the principles of laissez-faire economics.



ManhattanGMAT Sentence Correction Strategy Guide Subject Verb Agreement Copyright © 2009 MG Prep, Inc.



#### 2 errors

#### Identify the errors:

She requires that all of her children are to churn butter; resulting from this daily chore, the children have formidable biceps.

g



If I was the owner of our building, I would

demand, under threat of eviction, that you

should cease playing your djembe at all

g

2 errors



#### Verb Mood/Idioms

This question is about verb mood and idioms. "Requires that ... are" is incorrect. This clause is in the subjunctive mood, which uses the infinitive form of the verb without the "to" – in this case, simply "churn." "Resulting from" in the second clause is also incorrect – the correct idiom is "as a result of."

**One correction:** She requires that all of her children <u>churn</u> butter; <u>as a result of</u> this daily chore, the children have formidable biceps.

17



hours.

Identify the errors:

#### Verb Mood

This question is about the subjunctive mood. "If...was" is never correct ("If...would" is also incorrect). You must use the hypothetical subjunctive mood: "If...<u>were</u>." The command subjunctive is more commonly tested on the GMAT and is found in the second part of the sentence: "I would demand that you <u>should cease</u>." "Demand that you should" is incorrect on the GMAT. With "bossy verbs," use the bare form of the verb (the infinitive form of the verb without the "to").

**Correct:** If I <u>were</u> the owner of our building, I would demand, under threat of eviction, that you <u>cease</u> playing your djembe at all hours.



ManhattanGMAT Sentence Correction Strategy Guide Verb Mood Copyright © 2009 MG Prep, Inc.

3 errors

Identify the errors:

The committee told each of the disqualified candidates that their official policies prevented them from hiring them.

g



#### Pronouns

This question is about pronouns. "Committee" is singular; therefore, "their" and "them" do not match in number. "Each of the disqualified candidates" is also singular ("each" is singular, and "of the disqualified candidates" is simply a prepositional phrase); therefore, the second "them" does not match in number. Additionally, the use of "them" and "them" so close together is ambiguous.

**One correction:** The committee told each of the disqualified candidates that <u>its</u> official policies prevented <u>it</u> from hiring <u>him or her</u>.

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19



#### \_

1 error

Identify the error:

The nation's president would rather misappropriate foreign aid than actually attempting to help those in need.

## g

#### Parallelism

This question is about parallelism. Elements in the construction "rather X than Y" must be parallel.

**Correct:** The nation's president would rather misappropriate foreign aid than actually <u>attempt</u> to help those in need.



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4 errors

#### Identify the errors:

Joanne has the ability of walking on her hands, also her twin sister Kimberly has the ability to balance on a tightrope; I, however, try to hide my ability for juggling, lest others force me to demonstrate.

g



#### Idioms

This question is about idioms. "Ability of" and "ability for" are both wrong. "Ability to" is correct, but "can" is more concise and should be used where possible. Additionally, two independent clauses cannot be joined with "also."

**One correction:** Joanne <u>can walk</u> on her hands, <u>and</u> her twin sister Kimberly <u>can</u> balance on a tightrope; I, however, try to hide my ability <u>to juggle</u>, lest others force me to demonstrate.



#### 2 errors

Identify the errors:

Each of the athletes in the parade are carrying their nation's flag.

# g



#### Subject Verb Agreement/Pronouns

This sentence has a subject-verb error and a pronoun error. Although the sentence appears to be about athletes, the subject is actually the pronoun "each," which is singular. Therefore the verb "are" and the pronoun "their" do not match.

**One correction:** Each of the athletes in the parade <u>is</u> carrying <u>his or her</u> nation's flag.



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2 errors

Identify the errors:

The test was easy enough that I mastered it after a simple brain transplant. The transplant, however, was expensive to such a degree that I had to take out a second mortgage.

g



#### Idioms

This is a question about idioms. "Easy enough that" is incorrect. "Enough" must be followed by an infinitive or the preposition "for." Additionally, "expensive to such a degree as to" is incorrect and extremely wordy. It must be structured "so expensive that..."

**Correct:** The test was <u>easy enough for me to master</u> after a simple brain transplant. The transplant, however, was <u>so</u> <u>expensive</u> that I had to take out a second mortgage.

23



Identify the error:

The princess entered the spectacular arranged ballroom as a well-practiced band paid her a musical tribute.

# g

1 error



### Modifiers

This question is about adverbs (found under "Modifiers" in your SC Strategy Guide). An adverb is needed to modify an adjective. That is, if the ballroom were both spectacular and arranged, we would say "the spectacular, arranged ballroom" – but that doesn't make any sense. The word "spectacular" modifies the way the ballroom was "arranged" and should therefore be changed to "spectacularly." On the other hand, "well-practiced band" is correct, by the same logic; the adverb "well" modifies the adjective "practiced."

**Correct:** The princess entered the <u>spectacularly</u> arranged ballroom as a well-practiced band paid her a musical tribute.



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3 errors

Identify the errors:

My child has better hygiene than those of my sister; neither of her boys ever seem to wash their hands.

g



Identify the error:

Superintendent Jackson's administration is beginning to reform the school system, hiring new administrators, an increase in security, and overhauling the curriculum.

g



Subject-Verb Agreement/Pronouns

This question is about subject-verb agreement and pronouns. The pronoun "those" seems to be referring back to "child." Since "child" is singular, this is incorrect. Additionally, since "neither" is singular ("of her boys" is merely a prepositional phrase), the verb "seem" and the pronoun "their" are both incorrect.

**Correct:** My child has better hygiene than <u>my sister's children;</u> neither of her boys ever <u>seems</u> to wash <u>his</u> hands.





#### Parallelism

This problem is about parallelism. The administration is doing three things – hiring, *increasing*, and overhauling – all under the umbrella of "is beginning to reform the school system." To fix the sentence, we must make each item in the list match.

**Correct:** Superintendent Jackson's administration is beginning to reform the school system, hiring new administrators, <u>increasing</u> security, and overhauling the curriculum.



ManhattanGMAT Sentence Correction Strategy Guide Parallelism Copyright © 2009 MG Prep, Inc.

1 error

#### Identify the error:

The resort was advertised as a place where singles can meet one another and that offers an Olympic-sized swimming pool.

g



#### Parallelism

This question is about parallelism. Parallel clauses involving relative pronouns should start with the same relative pronoun ("where" and "that" do not match.)

**One correction:** The resort was advertised as a place where singles can meet one another and <u>where guests can swim in</u> an Olympic-sized swimming pool.



#### Identify the errors:

The iconoclastic fitness guru preaches three main principles: eat five apples per day, exercising before a two-hour brunch, and that you must burn calories by re-enacting action movies in your living room.

## g

2 errors

### SC

#### Parallelism

This question is about parallelism. Elements in a list must be parallel.

**One correction:** The iconoclastic fitness guru preaches three main principles: <u>eating</u> five apples per day, exercising before a two-hour brunch, <u>and burning</u> calories by re-enacting action movies in your living room.

**OR:** The iconoclastic fitness guru preaches three main principles: eat five apples per day, <u>exercise</u> before a two-hour brunch, <u>and burn</u> calories by re-enacting action movies in your living room.



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#### 2-4 errors

#### Identify the errors:

Being a pacifist, Katrin was not only upset he was being so militaristic, but, that he was torturing the captives.

## g



#### Idioms

This question is about idioms. First, the word "being" is usually suspect on the GMAT. The first "being" is definitely suspect; it can easily be removed. The second "being," which is before an adjective, is okay – "being militaristic" is a correct use of "being." The second idiom tested in this sentence is "<u>not only X but also Y.</u>" In this sentence, we have only "not only X <u>but</u> Y."

**One correction:** <u>A</u> pacifist, Katrin was <u>upset not only</u> that he was being so militaristic <u>but also</u> that he was torturing the captives.



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29



#### 2 errors

Identify the errors:

The United States are such an enticing locale to my wife's family that my father-in-law has decided to relocate their business to Minneapolis.

# g



#### Subject Verb Agreement/Pronouns

This question is about subject-verb agreement and pronouns. The United States, like all countries, is singular. "Family," a collective noun, is also singular. (Note: many collective nouns are considered plural in British English, so if you are more familiar with British than American English, you will have to adjust to this change for purposes of the GMAT.)

**One correction:** The United States <u>is</u> such an enticing locale to my wife's family that my father-in-law has decided to relocate <u>the family</u> business to Minneapolis. (Note: "relocate its business" is also an option, but "the family business" is very precise and clear.)



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2 errors

Identify the errors:

Mr. Kozlowski is no less talented a dancer as Justin Timberlake; however, unlike with Timberlake, Mr. Kozlowski lacks a certain celebrity quality.

g



Identify the error:

Although her shyness was keeping Sofia from asking Walter to dance, after consuming an energy-drink cocktail, she was able to do it.

## g



#### Idioms/Comparisons

This question is about idioms and comparisons. "No less talented *as*" is incorrect. It must be phrased "no less talented *than*." "Unlike with" is incorrect – the comparison of nouns must only use the word "unlike."

**Correct:**. Mr. Kozlowski is no less talented a dancer <u>than</u> Justin Timberlake; however, <u>unlike Timberlake</u>, Mr. Kozlowski lacks a certain celebrity quality.





#### Pronouns

This question is about pronouns. Be very suspicious of pronouns that appear late in a sentence. They often don't have a clear antecedent. "Do it" may only be used in a case in which "it" actually refers to a noun, such as in the case "Here is your *assignment*; I suggest that you do *it*."

**One correction**: Although her shyness was keeping Sofia from asking Walter to dance, after consuming an energy-drink cocktail, she was able to <u>ask him</u>. (Note: "...able to do so" would also be an option here.)



ManhattanGMAT Sentence Correction Strategy Guide Pronouns

#### 2 errors

Identify the errors:

Mastering Sanskrit is an often-frustrating journey where a student that is too impatient is likely to become discouraged.

g



#### Modifiers

This question is about modifiers, specifically those beginning with the relative pronoun "where." "Where" may only be used to describe actual places. A "journey," whether metaphorical or literal, is not a place. Furthermore, people may not be modified with "that" or "which"; only "who" is correct.

**One correction:** Mastering Sanskrit is an often-frustrating journey <u>during which</u> a student <u>who</u> is too impatient is likely to become discouraged.

33



### 2-4 errors

Identify the errors:

Because Agent M was believed by Agent Q to be trustworthy is the reason Agent Q allowed Agent M the conducting of the top secret mission alone.

## g

#### Idioms/Verb Voice

This question is about idioms and verb voice. "Because" and "the reason" serve the same purpose here; to use both is redundant and thus incorrect. "M was believed by Q" is passive voice and therefore suspect; use the active voice instead. "Allowed Agent M the conducting of" is unidiomatic.

**One correction:** Because <u>Agent Q believed that Agent M</u> <u>was trustworthy</u>, Agent Q allowed Agent M <u>to conduct</u> the top secret mission alone.



SC

ManhattanGMAT Sentence Correction Strategy Guide Idioms/Verb Voice Copyright © 2009 MG Prep, Inc.

#### 3 errors

SC

#### Identify the errors:

Since 1973, the record was broken eight times because of technology that advanced even more rapidly than commentators have predicted.

g



#### Verb Tense

This question is about verb tense. Using "since" necessitates the present perfect ("has been broken") to indicate an action that continues to occur over time. Similarly, since records are still being broken, technology is likely still advancing (thus we must use "has advanced"). Because the commentators' predictions occurred before the records were broken and are no longer in effect, the present perfect ("have predicted") is incorrect – past perfect or simple past must be used.

**One correction:** Since 1973, the record <u>has been</u> broken eight times because of technology that <u>has advanced</u> even more rapidly than commentators <u>had predicted</u> (or simply "predicted").



35



#### 2 errors

Identify the errors:

I intend not only to win but also break the world record; my opponent is neither a threat to me nor, quite frankly, intimidating to anyone.

## g

### SC

#### Parallelism

This question is about parallelism. Elements in a "not only...but also" construction, as well as elements in a "neither...nor" construction, must be parallel.

**Correct:** I intend not only to win but also <u>to break</u> the world record; my opponent is neither <u>threatening</u> to me nor, quite frankly, intimidating to anyone.



ManhattanGMAT Sentence Correction Strategy Guide Parallelism Copyright © 2009 MG Prep, Inc.

1 error

Identify the error:

On the commission is an expert in neuroscience and three members of a team that developed a promising new therapy for Parkinson's.

g



Identify the errors:

Peter thinks he can swim fast like Michael Phelps does; he has also claimed to be an unrecognized world champion of sports like sprinting and boxing.

## g



#### Subject Verb Agreement

This question is about subject-verb agreement. Note that this sentence has an inverted order: the verb "is" comes before the compound plural subject, "an expert in neuroscience and three members of a team." Ignore the prepositional phrases and flip the sentence (subject before verb) to spot the error: "An expert … and three members … is on the commission."

**Correct:** On the commission <u>are</u> an expert in neuroscience and three members of a team that developed a promising new therapy for Parkinson's.



37



#### Idioms/Comparisons

This question is about idioms and comparisons. "Like" compares nouns; "as" compares verbs or clauses. The "like" in the second sentence means "similar to but not including"—clearly the speaker means to include sprinting and boxing in the list of sports that Peter believes he has mastered.

**Correct:** Peter thinks he can swim <u>as fast as</u> Michael Phelps does; he has also claimed to be an unrecognized world champion of <u>such sports as</u> sprinting and boxing.



ManhattanGMAT Sentence Correction Strategy Guide Idioms/Comparisons Copyright © 2009 MG Prep, Inc.

#### 3 errors

Identify the errors:

Maribel proved herself deserving of the MVP award in girls' basketball, scored a record number of goals, became the first female player at the school to slam dunk in a championship game, and inspired her teammates immeasurably.

g



Identify the error:

Sandeep was told by his parents that if he helped his brother Rajeev get an "A" in trigonometry, he could go to the batting cage.

## g



#### Parallelism

This question is about parallelism. Elements in a list must be parallel. However, here we have a case of superficial parallelism. How many things did Maribel do? She did three things (scored, became, and inspired) all under the umbrella of *proving herself*. Only the last three must be parallel to one another, because they are subordinates to the main verb, "proved," and since they all describe "proved," an "–ing" ending is appropriate.

**Correct:** Maribel proved herself deserving of the MVP award in girls' basketball, <u>scoring</u> a record number of goals, <u>becoming</u> the first female player at the school to slam dunk in a championship game, and <u>inspiring</u> her teammates immeasurably.

39



#### Pronouns

This question is about pronouns. Although it seems apparent that a trip to the batting cages will be Sandeep's reward for a job well done, the "he" in the second part of the sentence could be referring to his brother Rajeev. With no clear antecedent, the pronoun is ambiguous, and therefore incorrectly used.

**Correct:** Sandeep was told by his parents that if he helped his brother Rajeev get an "A" in trigonometry, <u>Sandeep</u> could go to the batting cage.



ManhattanGMAT Sentence Correction Strategy Guide Pronouns

#### 0-4 errors

#### Identify the errors:

My mother considers my vegetarianism as an insult to her cooking – it seems like she is obsessed with convincing me to try the rack of lamb.

g



#### Idioms

This question is about idioms. "Consider X as Y" is incorrect (the "as" is unnecessary). It should be "consider X Y." "Seems like" followed by a clause is incorrect. "Seems that" or "seems as if" are both correct replacements.

**Correct:** My mother <u>considers my vegetarianism an insult</u> to her cooking - it seems <u>as if</u> she is obsessed with convincing me to try the rack of lamb.

41



#### 0-2 errors

Identify the errors:

Joe, who hasn't showered in days, smells bad—however, John, who is suffering from nasal congestion, smells so badly that Joe's odor doesn't bother him at all.

## g



#### Modifiers

This question is about adjectives and adverbs, and it's a little bit of a trick – however strange it may sound, this sentence is perfectly correct. Joe, who hasn't showered, smells <u>bad</u>. We are modifying a noun (Joe), so we want an adjective. John, who is stuffed up, smells <u>badly</u>. We are modifying a verb (smells), so we want an adverb. Both uses are correct, although they have very different meanings!

#### No correction.



ManhattanGMAT Sentence Correction Strategy Guide Modifiers

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#### 2-5 errors

Identify the errors:

The highly-advanced robot is proficient both in Japanese and English – and there is an expectation by us that the robot's maker should produce the new model only in limited quantities for maintaining the product's cachet.

g



#### Idioms

This question is about idioms. In the construction "both X and Y," X and Y need to take the same form (in this sentence, the word "in" is in the wrong place, thereby making X and Y unparallel.) "Expectation by" is incorrect, and the word "should" is also incorrect. We must use "expect that" and "will." "For maintaining" is incorrect here; "in order to maintain" is the intended meaning.

**Correct:** The highly-advanced robot is proficient in both Japanese and <u>English</u> – and <u>we expect that</u> the robot's maker <u>will</u> produce the new model only in limited quantities in order to maintain the product's cachet.

43



#### 2 errors

Identify the errors:

A number of my old classmates is coming to the reunion, but the number of impediments to my traveling to Arkansas next week are quite large.

## g



#### Subject Verb Agreement

This question is about subject-verb agreement. "A number of" is an idiomatic expression that takes a plural verb. "The number" is singular.

**Correct:** A number of my old classmates <u>are</u> coming to the reunion, but the number of impediments to my traveling to Arkansas next week <u>is</u> quite large.



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### 0-2 errors

#### Identify the error(s):

Alexander the Great remarked that conquering foreign states couldn't compare to the vanquishing of his enemies at home.

g



#### Parallelism

This question is about parallelism. Two items being compared to one another must be parallel. "Conquering," a simple gerund, cannot be made parallel to "the vanquishing of," a complex gerund.

**One correction:** Alexander the Great remarked that conquering foreign states couldn't compare to <u>vanquishing</u> his enemies at home.



45



#### Identify the errors:

Us attending the conference is dependent on our pilot receiving clearance to take off.

## g

2 errors

#### Verb Tense

This question is about gerunds and verb tense. In this sentence, "attending" and "receiving" are gerunds, and take possessive nouns and pronouns, <u>not</u> direct objects.

**Correct:** <u>Our</u> attending the conference is dependent on our <u>pilot's</u> receiving clearance to take off.



SC

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#### 2 errors

#### Identify the errors:

Despite having studied French for ten years, yet I cannot conjugate French verbs in any tense but the present; my most recent report card indicates me as an underachiever.

g



Identify the errors:

Once you show her the ropes, Josephina's natural skill as a salesperson will allow her to run the store by herself in no time.

## g



#### Idioms

This question is about idioms. The words "despite" and "yet" are redundant and therefore cannot be used. "Indicate as" is also incorrect. We must use "indicate that."

**Correct:** Despite having studied French for ten years, <u>I cannot</u> conjugate French verbs in any tense but the present; my most recent report card indicates <u>that I am</u> an underachiever. (Note that "yet" has been eliminated).





#### Modifiers

This question is about modifiers and pronouns. "Once you show her the ropes" is clearly about Josephina. However, "Josephina" (the noun) never appears in the sentence. "Josephina's" is a possessive noun modifying "skill." Thus, the modifier and the pronouns "her" and "herself" are wrong.

**One correction:** Once you show <u>Josephina</u> the ropes, <u>her</u> natural skill as a salesperson will allow her to run the store by herself in no time.



ManhattanGMAT Number Properties Strategy Guide Modifiers

#### 0-2 errors

Identify the error:

You can either increase your output or you can seek work elsewhere.

g



Identify the error:

Mr. Vega's mother noticed that a button was falling off his coat, so the coat was fixed by her.

## g



#### Parallelism

This question is about parallelism. Elements in an "either...or" construction must be parallel. The first "you" occurs prior to and is not part of the "either...or" construction. This "you" already applies to both elements in the construction, making the second "you" incorrect.

**One correction:** You can either increase your output or <u>seek</u> work elsewhere.



49



#### Verb Voice

This question is about verb voice. When possible you should keep verb voice consistent. The sentence begins in the active voice "Mr. Vega's mother noticed", and then switches to the passive voice "the coat was fixed by her" for no reason. Active voice is preferred.

**One correction:** Mr. Vega's mother noticed that a button was falling off his coat, so <u>she fixed the coat</u>.



ManhattanGMAT Sentence Correction Strategy Guide Verb Voice/Pronouns

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2 errors

#### Identify the errors:

Timmy only expresses affection for his dog, Lassie; as a result, Lassie only eats top-shelf dog food.

g



#### Idioms

This question is about idioms. "Only" should be placed just before the word or words it is meant to modify. The first sentence seems to be saying that the only thing Timmy ever does is express his affection for Lassie. Similarly, the second sentence seems to be saying that Lassie never does anything other than eat top-shelf dog food.

**Correct:**. Timmy expresses affection <u>only</u> for his dog, Lassie; as a result, Lassie eats <u>only</u> top-shelf dog food.





#### Identify the error:

The ubiquity of the personal music player since the start of the millennium provides users with a distraction on long commutes and can now hold thousands of songs.

# g

1 error

### SC

#### Subject Verb Agreement

This question is about subject-verb agreement. The subject "ubiquity" and the verb "provides" are both singular, so the pair matches in number. However, a subject-verb pair must also be *logical.* It is the personal music player, not the ubiquity, that "provides users with a distraction on long commutes and can now hold thousands of songs."

**One correction**: <u>The personal music player, ubiquitous since the</u> <u>start of the millennium</u>, provides users with a distraction on long commutes and can now hold thousands of songs.



ManhattanGMAT Sentence Correction Strategy Guide Subject Verb Agreement Copyright © 2009 MG Prep, Inc.

#### 0-2 errors

#### Identify the errors:

My boss is likely that she will take over the eastern division, even though she asked for the northern division.

# g



#### Idioms

This question is about idioms. A person cannot be "likely that", only "likely to" (although "likely that" can precede a clause, such as in "It is likely that it will rain"). Also, the correct construction is "not X but Y" or "not X but rather Y," where X and Y must take the same form.

**Correct:** My boss is <u>likely to take over</u> the eastern division, even though she asked for the northern division.



#### 0-2 errors

Identify the error:

A scant 150 words long, the Gettysburg Address was written by Abraham Lincoln on the train to Gettysburg; this fact was discovered by Meg while doing research for her book report.

## g



#### Verb Voice

This question is about the passive voice. The passive voice is used twice here: "the Gettysburg Address was written by Abraham Lincoln" and "This fact was discovered by me." The use of the passive voice in the first sentence is required because the modifier "Coming in at a scant 150 words" modifies the Gettysburg Address, not Lincoln. The second use of the passive voice, however, is awkward and unnecessary.

**Correct:** A scant 150 words long, the Gettysburg Address was written by Abraham Lincoln on the train to Gettysburg; <u>Meg</u> <u>discovered this fact</u> while doing research for her book report.



<u>ManhattanGMAT Sentence Correction Strategy Guide</u> Verb Voice

#### 0-2 errors

#### Identify the errors:

Among the dog's offenses are a refusal to eat anything but expensive rib eye steak and a penchant for attacking postal workers, neighbors and relatives, including my nephew and grandmother.

g



### Identify the errors:

In spite of Charlie's protest, his friends including his closest companion Rafael insisted that Charlie consider outside aid in dealing with his eating disorder.

## g



#### Parallelism

This question is about parallelism. The original sentence is correct as written. There are actually three lists within this sentence, which could be outlined as follows:

#### <u>This Dog's Offenses:</u> L. A refusal to eat

I. A refusal to eat
II. A penchant for attacking

a. Postal workers
c. Neighbors
b. Relatives
1. Nephew
2. Grandmother

#### No Correction.

ManhattanGMAT Sentence Correction Strategy Guide Parallelism Copyright © 2009 MG Prep, Inc. SC

#### Pronouns

0-2 errors

This question is about pronouns. It may seem awkward to keep repeating "Charlie" throughout the sentence, rather than using pronouns to refer to him. However, the first instance of "Charlie" is in a possessive phrase, so it cannot act as an antecedent. The use of "his" in the phrase "his friends" is okay, because only possessive pronouns can refer to nouns that are part of a possessive clause. The original sentence is correct!

#### No Correction.



55

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