WORD TRANSLATIONS

Math Strategy Guide

This comprehensive guide analyzes the GMAT's complex word problems and provides structured frameworks for attacking each question type. Master the art of translating challenging word problems into organized data.

Word Translations GMAT Strategy Guide, Fourth Edition

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Dear Student,

Thank you for picking up one of the ManhattanGMAT Strategy Guides—we hope that it refreshes your memory of the junior-high math that you haven't used in years. Maybe it will even teach you a new thing or two.

As with most accomplishments, there were many people involved in the various iterations of the book that you're holding. First and foremost is Zeke Vanderhoek, the founder of ManhattanGMAT. Zeke was a lone tutor in New York when he started the Company in 2000. Now, nine years later, MGMAT has Instructors and offices nationwide, and the Company contributes to the studies and successes of thousands of students each year.

Our 4th Edition Strategy Guides are based on the continuing experiences of our Instructors and our students. We owe much of these latest editions to the insight provided by our students. On the Company side, we are indebted to many of our Instructors, including but not limited to Josh Braslow, Dan Gonzalez, Mike Kim, Stacey Koprince, Ben Ku, Jadran Lee, David Mahler, Ron Purewal, Tate Shafer, Emily Sledge, and of course Chris Ryan, the Company's Lead Instructor and Director of Curriculum Development.

At ManhattanGMAT, we continually aspire to provide the best Instructors and resources possible. We hope that you'll find our dedication manifest in this book. If you have any comments or questions, please e-mail me at andrew.yang@manhattangmat.com. I'll be sure that your comments reach Chris and the rest of the team—and I'll read them too.

Best of luck in preparing for the GMAT!

Sincerely,

Andrew Yang
Chief Executive Officer
Manhattan GMAT

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PART I: GENERAL

This part of the book covers both basic and intermediate topics within *Word Translations*. Complete Part I before moving on to Part II: Advanced.

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Chapter 1 *of* WORD TRANSLATIONS

ALGEBRAIC TRANSLATIONS

In This Chapter . . .

- Algebraic Translations
- Translating Words Correctly
- Using Charts to Organize Variables
- Prices and Quantities
- Hidden Constraints

Algebraic Translations

To solve many word problems on the GMAT, you must be able to translate English into algebra. You assign variables to represent unknown quantities. Then you write equations to state relationships between the unknowns and any known values. Once you have written one or more algebraic equations to represent a problem, you solve them to find any missing information. Consider the following example:

> A candy company sells premium chocolates at $5 per pound and regular chocolates at $4 per pound. If Barrett buys a 7-pound box of chocolates that costs him $31, how many pounds of premium chocolates are in the box?

Be sure to make a note of what each variable represents. If you can, use meaningful letters as variable names.

Step 1: Assign variables.
Make up letters to represent unknown quantities, so you can set up equations. Sometimes, the problem has already named variables for you, but in many cases you must take this step yourself—and you cannot proceed without doing so.

Which quantities? Choose the most basic unknowns. Also consider the "Ultimate Unknown"—what the problem is directly asking for. In the problem above, the quantities to assign variables to are the number of pounds of premium chocolates and the number of pounds of regular chocolates.

Which letters? Choose different letters, of course. Choose meaningful letters, if you can. If you use x and y, you might forget which stands for which type of chocolate. For this problem, you could make the following assignments (and actually write them on your scrap paper):

p = pounds of premium chocolates
r = pounds of regular chocolates

Do not forget the "pounds" unit, or you might think you are *counting* the chocolates, as you might in a different problem. Alternatively, you could write "p = weight of premium chocolates (pounds)." Also, generally avoid creating subscripts—they can make equations look needlessly complex. But if you have several quantities, subscripts might be useful. For instance, if you have to keep track of the male and female populations of two towns, you could write m_1, m_2, f_1, and f_2. Some Algebraic Translation problems give you variables with subscripts, so be ready to work with them.

In the example problem, p is the Ultimate Unknown. A good way to remind yourself is to write "p = ?" on your paper, so that you never forget what you are ultimately looking for.

Try to minimize the number of variables. Often you can save work later if you just name one variable at first and use it to express more than one quantity before you name a second variable. How can you use a variable to express more than one quantity? Make use of a relationship given in the problem.

For instance, in the problem above, we know a simple relationship between the premium and the regular chocolates: their weights must add up to 7 pounds. So, if we know one of the weights, we can subtract it from 7 to get the other weight. Thus, we could have made these assignments:

p = pounds of premium chocolates
$7 - p$ = pounds of regular chocolates

Or you might have written both p and r at first, but then you could immediately make use of the relationship $p + r = 7$ to write $r = 7 - p$ and get rid of r.

Step 2: Write equation(s).
If you are not sure how to construct the equation, begin by expressing a relationship between the unknowns and the known values in **words**. For example, you might say:

"The total cost of the box is equal to the cost of the premium chocolates plus the cost of the regular chocolates."

Most algebraic translation problems involve only the 4 simple arithmetic processes: addition, subtraction, multiplication, and division. Look for totals, differences, products and ratios.

Or you might even write down a "Word Equation" as an intermediate step:
"Total Cost of Box = Cost of Premiums + Cost of Regulars"

Then, translate the verbal relationship into mathematical symbols. Use another relationship, *Total Cost* = *Unit Price* × *Quantity,* to write the terms on the right hand side. For instance, the "Cost of Premiums" in dollars = ($5 per pound)($p$ pounds) = $5p$.

$$31 = 5p + 4(7 - p)$$

The total cost of the box — is equal to — the cost of the premium chocolates — plus — the cost of the regular chocolates

Many word problems, including this one, require a little basic background knowledge to complete the translation to algebra. Here, to write the expressions $5p$ and $4(7 - p)$, you must understand that *Total Cost* = *Unit Price* × *Quantity*. In this particular problem, the quantities are weights, measured in pounds, and the unit prices are in dollars per pound.

Although the GMAT requires little factual knowledge, it will assume that you have mastered the following relationships:

- Total Cost = Unit Price × Quantity purchased
- Total Sales or Revenue = Unit Price × Quantity sold
- Profit = Revenue − Cost
- Unit Profit = Sale Price − Unit Cost or Sale Price = Unit Cost + Markup
- Total Earnings ($) = Wage Rate ($/hour) × Hours worked

Finally, note that you need to express some relationships as inequalities, not as equations.

Step 3: Solve algebraically.

$$31 = 5p + 4(7 - p)$$
$$31 = 5p + 28 - 4p$$
$$3 = p$$

Step 4: Evaluate the algebraic solution in the context of the problem.
Once you solve for the unknown, look back at the problem and make sure you answer the question asked. In this problem, we are asked for the number of pounds of premium chocolates. Notice that we wisely chose our variable p to represent this Ultimate Unknown. This way, once we have solved for p, we are finished. If you use two variables, p and r, and accidentally solve for r, you might choose 4 as your answer.

Translating Words Correctly

Avoid writing relationships backwards.

If You See…	Write	Not
"A is half the size of B"	✓ $A = \frac{1}{2}B$	✗ $B = \frac{1}{2}A$
"A is 5 less than B"	✓ $A = B - 5$	✗ $A = 5 - B$
"A is less than B"	✓ $A < B$	✗ $A > B$
"Jane bought twice as many apples as bananas"	✓ $A = 2B$	✗ $2A = B$

Be ready to insert simple test numbers to make sure that your translation is correct.

Quickly check your translation with easy numbers.

For the last example above, you might think the following:

> "Jane bought twice as many apples as bananas. More apples than bananas. Say she buys 5 bananas. She buys twice as many apples—that's 10 apples. Makes sense. So the equation is Apples equals 2 times Bananas, or $A = 2B$, not the other way around."

These numbers do not have to satisfy any other conditions of the problem. Use these "quick picks" only to test the form of your translation.

Write an unknown percent as a variable divided by 100.

If You See…	Write	Not
"P is X percent of Q"	✓ $P = \frac{X}{100}Q$ or $\frac{P}{Q} = \frac{X}{100}$	✗ $P = X\%\,Q$ (cannot be manipulated)

Translate bulk discounts and similar relationships carefully.

If You See…	Write	Not
"Pay \$10 per CD for the first 2 CDs, then \$7 per additional CD"	✓ n = # of CDs bought T = total amount paid (\$) $T = \$10 \times 2 + \$7 \times (n - 2)$ (assuming $n > 2$)	✗ $T = \$10 \times 2 + \$7 \times n$

Always pay attention to the *meaning* of the sentence you are translating!

The age chart does not relate the ages of the individuals. It simply helps you to assign variables you can use to write equations.

Using Charts to Organize Variables

When an algebraic translation problem involves several quantities and multiple relationships, it is often a good idea to make a chart or a table to organize the information.

One type of algebraic translation that appears on the GMAT is the "age problem." Age problems ask you to find the age of an individual at a certain point in time, given some information about other people's ages at other times.

Complicated age problems can be effectively solved with an Age Chart, which puts people in rows and times in columns. Such a chart helps you keep track of one person's age at different times (look at a row), as well as several ages at one time (look at a column).

> 8 years ago, George was half as old as Sarah. Sarah is now 20 years older than George. How old will George be 10 years from now?

Step 1: Assign variables.
Set up an Age Chart to help you keep track of the quantities. Put the different people in rows and the different times in columns, as shown below. Then assign variables. You could use two variables (G and S), or you could use just one variable (G) and represent Sarah's age right away as $G + 20$, since we are told that Sarah is now 20 years older than George. We will use the second approach. Either way, always use the variables to indicate the age of each person *now*. Fill in the other columns by adding or subtracting time from the "now" column (for instance, subtract 8 to get the "8 years ago" column). Also note the Ultimate Unknown with a question mark.

	8 years ago	Now	10 years from now
George	$G - 8$	G	$G + 10 = ?$
Sarah	$G + 12$	$G + 20$	$G + 30$

Step 2: Write equation(s).
Write equations that relate the individuals' ages together. According to this problem, 8 years ago, George was half as old as Sarah. Using the age expressions in the "8 years ago" column, we can write the following equation:

$$G - 8 = \frac{G + 12}{2} \qquad \text{which can be rewritten as} \qquad 2G - 16 = G + 12$$

Step 3: Solve algebraically.

$$2G - 16 = G + 12$$
$$G = 28$$

Step 4: Evaluate the algebraic solution in the context of the problem.
In this problem, we are asked to find George's age in 10 years. Since George is now 28 years old, he will be 38 in 10 years. The answer is 38 years.

Note that if we had used two variables, G and S, we might have set the table up slightly faster, but then we would have had to solve a system of 2 equations and 2 unknowns.

In a typical Price–Quantity problem, you have two relationships. The quantities sum to a total, and the monetary values sum to a total.

Prices and Quantities

Many GMAT word problems involve the total price or value of a mixed set of goods. On such problems, you should be able to write two different types of equations right away.

1. Relate the *quantities* or numbers of different goods: Sum of these numbers = Total.
2. Relate the total *values* of the goods (or their total cost, or the revenue from their sale):
   Money spent on one good = Price × Quantity.
   Sum of money spent on all goods = Total Value.

The following example could be the prompt of a Data Sufficiency problem:

> Paul has twenty-five transit cards, each worth either \$5, \$3, or \$1.50. What is the total monetary value of all of Paul's transit cards?

Step 1. Define variables
There are three quantities in the problem, so the most obvious way to proceed is to designate a separate variable for each quantity:

x = number of \$5 transit cards
y = number of \$3 transit cards
z = number of \$1.50 transit cards

Alternatively, you could use the given *relationship* between the three quantities (they sum to 25) to reduce the number of variables from three to two:

number of \$5 transit cards = x
number of \$3 transit cards = y
number of \$1.50 transit cards = $25 - x - y$ or $25 - (x + y)$

Note that in both cases, the Ultimate Unknown (the total value of the cards) is *not* given a variable name. This total value is not a simple quantity; we will express it *in terms of* the variables we have defined.

Step 2. Write equations
If you use three variables, then you should write two equations. One equation relates the *quantities* or numbers of different transit cards; the other relates the *values* of the cards.

Numbers of cards: $x + y + z = 25$
Values of cards: $5x + 3y + 1.50z = ?$ (this is the Ultimate Unknown for the problem)

If you have trouble writing these equations, you can use a chart or a table to help you. The **columns** of the table are *Unit Price*, *Quantity*, and *Total Value* (with *Unit Price* × *Quantity* = *Total Value*). The **rows** correspond to the different types of items in the problem, with one additional row for *Total*.

In the *Quantity* and *Total Value* columns, but not in the *Unit Price* column, the individual rows sum to give the quantity in the *Total* row. Note that *Total Value* is a quantity of money (usually dollars), corresponding either to *Total Revenue*, *Total Cost*, or even *Total Profit*, depending on the problem's wording.

You can use a table to organize your approach to a Price–Quantity problem. However, if you learn to write the equations directly, you will save time.

For this type of problem, you can save time by writing the equations directly. But feel free to use a table.

	Unit Price	×	*Quantity*	=	*Total Value*
\$5 cards	5	×	x	=	$5x$
\$3 cards	3	×	y	=	$3y$
\$1.50 cards	1.5	×	z	=	$1.5z$
Total	—		25		?

Notice that the numbers in the second and third columns of the table (*Quantity* and *Total Value*) can be added up to make a meaningful total, but the numbers in the first column (*Unit Price*) do not add up in any meaningful way.

If you use the two-variable approach, you do not need to write an equation relating the *numbers* of transit cards, because you have already used that relationship to write the expression for the number of \$1.50 cards (as $25 - x - y$). Therefore, you only need to write the equation to sum up the values of the cards.

Values of cards: $5x + 3y + 1.50(25 - x - y) = ?$
Simplify ➔ $3.5x + 1.5y + 37.5 = ?$

Here is the corresponding table:

	Unit Value	×	*Quantity*	=	*Total Value*
\$5 cards	5	×	x	=	$5x$
\$3 cards	3	×	y	=	$3y$
\$1.50 cards	1.5	×	$25 - x - y$	=	$1.5(25 - x - y)$
Total	—		25		?

All of the work so far has come just from the *prompt* of a Data Sufficiency question—you have not even seen statements (1) and (2) yet! But this work is worth the time and energy. In general, you should rephrase and interpret a Data Sufficiency question prompt as much as you can before you begin to work with the statements.

Hidden Constraints

Notice that in the previous problem, there is a **hidden constraint** on the possible quantities of cards (x, y, and either z or $25 - x - y$). Since each card is a physical, countable object, you can only have a **whole number** of each type of card. Whole numbers are the integers 0, 1, 2, and so on. So you can have 1 card, 2 cards, 3 cards, etc., and even 0 cards, but you cannot have fractional cards or negative cards.

As a result of this implied "whole number" constraint, you actually have more information than you might think. Thus, on a Data Sufficiency problem, you may be able to answer the question with less information from the statements.

As an extreme example, imagine that the question is "What is x?" and that statement (1) reads "$1.9 < x < 2.2$". If some constraint (hidden or not) restricts x to whole-number values, then statement (1) is sufficient to answer the question: x must equal 2. On the other hand, without constraints on x, statement (1) is not sufficient to determine what x is.

When a variable indicates how many objects there are, it must be a whole number.

In general, if you have a whole number constraint on a Data Sufficiency problem, you should suspect that you can answer the question with very little information. This pattern is not a hard-and-fast rule, but it can guide you in a pinch.

Recognizing a hidden constraint can be useful, not only on Data Sufficiency problems, but also on certain Problem Solving problems. Consider the following example:

> If Kelly received 1/3 more votes than Mike in a student election, which of the following could have been the total number of votes cast for the two candidates?
>
> (A) 12 (B) 13 (C) 14 (D) 15 (E) 16

Let M be the number of votes cast for Mike. Then Kelly received $M + (1/3)M$, or $(4/3)M$ votes. The total number of votes cast was therefore "votes for Mike" plus "votes for Kelly," or $M + (4/3)M$. This quantity equals $(7/3)M$, or $7M/3$.

Because M is a number of votes, it cannot be a fraction—specifically, not a fraction with a 7 in the denominator. Therefore, the 7 in the expression $7M/3$ cannot be cancelled out. As a result, the total number of votes cast must be a multiple of 7. Among the answer choices, the only multiple of 7 is 14, so the correct answer is (**C**).

Another way to solve this problem is this: the number of votes cast for Mike (M) must be a multiple of 3, since the total number of votes is a whole number. So $M = 3, 6, 9$, etc. Kelly received 1/3 more votes, so the number of votes she received is 4, 8, 12, etc. Thus the total number of votes is 7, 14, 21, etc.

When you have a whole number, you can also use a table to generate, organize, and eliminate possibilities. Consider the following problem:

> A store sells erasers for \$0.23 each and pencils for \$0.11 each. If Jessica buys both erasers and pencils from the store for a total of \$1.70, what total number of erasers and pencils did she buy?

Let E represent the number of erasers Jessica bought. Likewise, let P be the number of pencils she bought. Then we can write an equation for her total purchase. Switch over to cents right away to avoid decimals.

$$23E + 11P = 170$$

If E and P did not have to be integers, there would be no way to solve for a single result. However, we know that there is an answer to the problem, and so there must be a set of integers E and P satisfying the equation. First, rearrange the equation to solve for P:

$$P = \frac{170 - 23E}{11}$$

To solve algebra problems that have integer constraints, test possible values systematically in a table.

Since P must be an integer, we know that $170 - 23E$ must be divisible by 11. Set up a table to test possibilities, starting at an easy number (E = 0).

E	$P = \frac{170 - 23E}{11}$	Works?
0	$P = 170/11$	No
1	$P = 147/11$	No
2	$P = 124/11$	No
3	$P = 101/11$	No
4	$P = 78/11$	No
5	$P = 55/11 = 5$	Yes

Thus, the answer to the question is $E + P = 5 + 5 = 10$.

In this problem, the possibilities for E and P are constrained not only to integer values but in fact to positive values (since we are told that Jessica buys both items). Thus, we could have started at $E = 1$. We can also see that as E increases, P decreases, so there is a finite number of possibilities to check before P reaches zero.

Not every unknown quantity related to real objects is restricted to whole numbers. Many physical measurements, such as weights, times, or speeds, can be any positive number, not necessarily integers. A few quantities can even be negative (e.g., temperatures, x- or y-coordinates). Think about what is being measured or counted, and you will recognize whether a hidden constraint applies.

POSITIVE CONSTRAINTS = POSSIBLE ALGEBRA

When all the quantities are positive in a problem, whether or not they are integers, you should realize that **certain algebraic manipulations are safe to perform** and that they only generate one result. This realization can spell the difference between success and failure on many Data Sufficiency problems.

Study the following lists:

1. Dropping Negative Solutions of Equations

Manipulation	If You Know…	And You Know…	Then You Know…
Square rooting	$x^2 = 16$	$x > 0$	$x = 4$
Solving general quadratics	$x^2 + x - 6 = 0$ $(x + 3)(x - 2) = 0$	$x > 0$	$x = 2$

When all variables are positive, you can perform certain manipulations safely. Know these manipulations!

2. Dropping Negative Possibilities with Inequalities

Manipulation	If You Know…	And You Know…	Then You Know…
Multiplying by a variable	$\frac{x}{y} < 1$	$y > 0$	$x < y$
Cross-multiplying	$\frac{x}{y} < \frac{y}{x}$	$x > 0$ $y > 0$	$x^2 < y^2$
Dividing by a variable	Question: "Is $0.4x > 0.3x$?"	$x > 0$	Question becomes "Is $0.4 > 0.3$?" (Answer is yes)
Taking reciprocals and flipping the sign	$x < y$	$x > 0$ $y > 0$	$\frac{1}{x} > \frac{1}{y}$
Multiplying two inequalities (but NOT dividing them!)	$x < y$ $z < w$	$x, y, z, w > 0$	$xz < yw$
Squaring an inequality	$x < y$	$x > 0$ $y > 0$	$x^2 < y^2$
Unsquaring an inequality	$x < y$	$x > 0$ $y > 0$	$\sqrt{x} < \sqrt{y}$

Problem Set

Solve the following problems with the four-step method outlined in this section.

1. John is 20 years older than Brian. 12 years ago, John was twice as old as Brian. How old is Brian?

2. Mrs. Miller has two dogs, Jackie and Stella, who weigh a total of 75 pounds. If Stella weighs 15 pounds less than twice Jackie's weight, how much does Stella weigh?

3. Caleb spends $72.50 on 50 hamburgers for the marching band. If single burgers cost $1.00 each and double burgers cost $1.50 each, how many double burgers did he buy?

4. Abigail is 4 times as old as Bonnie. In 6 years, Bonnie will be twice as old as Candice. If, 4 years from now, Abigail will be 36 years old, how old will Candice be in 6 years?

5. United Telephone charges a base rate of $10.00 for service, plus an additional charge of $0.25 per minute. Atlantic Call charges a base rate of $12.00 for service, plus an additional charge of $0.20 per minute. For what number of minutes would the bills for each telephone company be the same?

6. Ross is 3 times as old as Sam, and Sam is 3 years older than Tina. 2 years from now, Tina will drink from the Fountain of Youth, which will cut her age in half. If after drinking from the Fountain, Tina is 16 years old, how old is Ross right now?

7. Carina has 100 ounces of coffee divided into 5- and 10-ounce packages. If she has 2 more 5-ounce packages than 10-ounce packages, how many 10-ounce packages does she have?

8. Carla cuts a 70-inch piece of ribbon into 2 pieces. If the first piece is five inches more than one fourth as long as the second piece, how long is the longer piece of ribbon?

9. In a used car lot, there are 3 times as many red cars as green cars. If tomorrow 12 green cars are sold and 3 red cars are added, then there will be 6 times as many red cars as green cars. How many green cars are currently in the lot?

1. **32:** Use an age chart to assign variables. Represent Brian's age now with b. Then John's age now is $b + 20$.

	12 years ago	Now
John	$b + 8$	$b + 20$
Brian	$b - 12$	$b = ?$

Subtract 12 from the "now" column to get the "12 years ago" column.

Then write an equation to represent the remaining information: 12 years ago, John was twice as old as Brian. Solve for b:

$$b + 8 = 2(b - 12)$$
$$b + 8 = 2b - 24$$
$$32 = b$$

You could also solve this problem by inspection. John is 20 years older than Brian. We also need John to be *twice* Brian's age at a particular point in time. Since John is always 20 years older, then he must be 40 years old at that time (and Brian must be 20 years old). This point in time was 12 years ago, so Brian is now 32 years old.

2. **45 pounds:**
Let j = Jackie's weight, and let s = Stella's weight. Stella's weight is the Ultimate Unknown: $s = ?$

The two dogs weigh a total of 75 pounds.
$$j + s = 75$$

Stella weighs 15 pounds less than twice Jackie's weight.
$$s = 2j - 15$$

Combine the two equations by substituting the value for s from equation (2) into equation (1).
$$j + (2j - 15) = 75$$
$$3j - 15 = 75$$
$$3j = 90$$
$$j = 30$$

Find Stella's weight by substituting Jackie's weight into equation (1).
$$30 + s = 75$$
$$s = 45$$

3. **45 double burgers:**
Let s = the number of single burgers purchased
Let d = the number of double burgers purchased

Caleb bought 50 burgers:
$$s + d = 50$$

Caleb spent \$72.50 in all:
$$s + 1.5d = 72.50$$

Combine the two equations by subtracting equation (1) from equation (2).
$$\begin{array}{r} s + 1.5d = 72.50 \\ -(s + d = 50) \\ \hline 0.5d = 22.5 \\ d = 45 \end{array}$$