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# NUMBER PROPERTIES

## Math Strategy Guide

This foundational guide provides a comprehensive analysis of the properties and rules of integers tested on the GMAT. Learn, practice, and master everything from prime products to perfect squares.

Number Properties GMAT Strategy Guide, Fourth Edition

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May 1st, 2009

Dear Student,

Thank you for picking up one of the Manhattan GMAT Strategy Guides—we hope that it refreshes your memory of the junior-high math that you haven't used in years. Maybe it will even teach you a new thing or two.

As with most accomplishments, there were many people involved in the various iterations of the book that you're holding. First and foremost is Zeke Vanderhoek, the founder of Manhattan GMAT. Zeke was a lone tutor in New York when he started the Company in 2000. Now, nine years later, MGMAT has Instructors and offices nationwide, and the Company contributes to the studies and successes of thousands of students each year.

Our 4th Edition Strategy Guides are based on the continuing experiences of our Instructors and our students. We owe much of these latest editions to the insight provided by our students. On the Company side, we are indebted to many of our Instructors, including but not limited to Josh Braslow, Dan Gonzalez, Mike Kim, Stacey Koprince, Ben Ku, Jadran Lee, David Mahler, Ron Purewal, Tate Shafer, Emily Sledge, and of course Chris Ryan, the Company's Lead Instructor and Director of Curriculum Development.

At Manhattan GMAT, we continually aspire to provide the best Instructors and resources possible. We hope that you'll find our dedication manifest in this book. If you have any comments or questions, please e-mail me at [andrew.yang@manhattangmat.com](mailto:andrew.yang@manhattangmat.com). I'll be sure that your comments reach Chris and the rest of the team—and I'll read them too.

Best of luck in preparing for the GMAT!

Sincerely,

Andrew Yang  
Chief Executive Officer  
Manhattan GMAT

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GENERAL

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# PART I: GENERAL

This part of the book covers both basic and intermediate topics within *Number Properties*. Complete Part I before moving on to Part II: Advanced.

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## **Chapter 1** *of* NUMBER PROPERTIES

## DIVISIBILITY & PRIMES

# In This Chapter . . .

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- Integers
- Arithmetic Rules
- Rules of Divisibility by Certain Integers
- Factors and Multiples
- Fewer Factors, More Multiples
- Divisibility and Addition/Subtraction
- Primes
- Prime Factorization
- Factor Foundation Rule
- The Prime Box
- Greatest Common Factor and Least Common Multiple
- Remainders

## INTEGERS

Integers are “whole” numbers, such as 0, 1, 2, and 3, that have no fractional part. Integers can be positive (1, 2, 3...), negative (-1, -2, -3...), or the number 0.

The GMAT uses the term integer to mean a non-fraction or a non-decimal. The special properties of integers form the basis of most Number Properties problems on the GMAT.

### Arithmetic Rules

Most arithmetic operations on integers will always result in an integer. For example:

$4 + 5 = 9$	$(-2) + 1 = -1$	The sum of two integers is always an integer.
$4 - 5 = -1$	$(-2) - (-3) = 1$	The difference of two integers is always an integer.
$4 \times 5 = 20$	$(-2) \times 3 = -6$	The product of two integers is always an integer.

However, division is different. Sometimes the result is an integer, and sometimes it is not:

$8 \div 2 = 4$ , but $2 \div 8 = \frac{1}{4}$	The result of dividing two integers is SOMETIMES an integer.
$(-8) \div 4 = -2$ , but $(-8) \div (-6) = \frac{4}{3}$	(This result is called the <b>quotient</b> .)

An integer is said to be **divisible** by another number if the integer can be divided by that number with an integer result (meaning that there is no remainder).

For example, 21 is divisible by 3 because when 21 is divided by 3, an integer is the result ( $21 \div 3 = 7$ ). However, 21 is not divisible by 4 because when 21 is divided by 4, a non-integer is the result ( $21 \div 4 = 5.25$ ).

Alternatively, we can say that 21 is divisible by 3 because 21 divided by 3 yields 7 with zero remainder. On the other hand, 21 is not divisible by 4 because 21 divided by 4 yields 5 with a remainder of 1.

Here are some more examples:

$8 \div 2 = 4$	Therefore, 8 is divisible by 2.
$2 \div 8 = 0.25$	We can also say that 2 is a <b>divisor</b> or <b>factor</b> of 8.
$(-6) \div 2 = -3$	Therefore, 2 is NOT divisible by 8.
$(-6) \div (-4) = 1.5$	Therefore, -6 is divisible by 2.
	Therefore, -6 is NOT divisible by -4.

Divisibility questions test your knowledge whether division of integers results in an integer.

## Rules of Divisibility by Certain Integers

The Divisibility Rules are important shortcuts to determine whether an integer is divisible by 2, 3, 4, 5, 6, 8, 9, and 10.

An integer is divisible by:

### **2 if the integer is EVEN.**

12 is divisible by 2, but 13 is not. Integers that are divisible by 2 are called “even” and integers that are not are called “odd.” You can tell whether a number is even by checking to see whether the units (ones) digit is 0, 2, 4, 6, or 8. Thus, 1,234,567 is odd, because 7 is odd, whereas 2,345,678 is even, because 8 is even.

### **3 if the SUM of the integer’s DIGITS is divisible by 3.**

72 is divisible by 3 because the sum of its digits is 9, which is divisible by 3. By contrast, 83 is not divisible by 3, because the sum of its digits is 11, which is not divisible by 3.

### **4 if the integer is divisible by 2 TWICE, or if the LAST TWO digits are divisible by 4.**

28 is divisible by 4 because you can divide it by 2 twice and get an integer result ( $28 \div 2 = 14$ , and  $14 \div 2 = 7$ ). For larger numbers, check only the last two digits. For example, 23,456 is divisible by 4 because 56 is divisible by 4, but 25,678 is not divisible by 4 because 78 is not divisible by 4.

### **5 if the integer ends in 0 or 5.**

75 and 80 are divisible by 5, but 77 and 83 are not.

### **6 if the integer is divisible by BOTH 2 and 3.**

48 is divisible by 6 since it is divisible by 2 (it ends with an 8, which is even) AND by 3 ( $4 + 8 = 12$ , which is divisible by 3).

### **8 if the integer is divisible by 2 THREE TIMES, or if the LAST THREE digits are divisible by 8.**

32 is divisible by 8 since you can divide it by 2 three times and get an integer result ( $32 \div 2 = 16$ ,  $16 \div 2 = 8$ , and  $8 \div 2 = 4$ ). For larger numbers, check only the last 3 digits. For example, 23,456 is divisible by 8 because 456 is divisible by 8, whereas 23,556 is not divisible by 8 because 556 is not divisible by 8.

### **9 if the SUM of the integer’s DIGITS is divisible by 9.**

4,185 is divisible by 9 since the sum of its digits is 18, which is divisible by 9. By contrast, 3,459 is not divisible by 9, because the sum of its digits is 21, which is not divisible by 9.

### **10 if the integer ends in 0.**

670 is divisible by 10, but 675 is not.

The GMAT can also test these divisibility rules in reverse. For example, if you are told that a number has a ones digit equal to 0, you can infer that that number is divisible by 10. Similarly, if you are told that the sum of the digits of  $x$  is equal to 21, you can infer that  $x$  is divisible by 3 but NOT by 9.

Note also that there is no rule listed for divisibility by 7. The simplest way to check for divisibility by 7, or by any other number not found in this list, is to perform long division.

It is a good idea to memorize the rules for divisibility by 2, 3, 4, 5, 6, 8, 9 and 10.

## Factors and Multiples

Factors and Multiples are essentially opposite terms.

A **factor** is a positive integer that divides evenly into an integer. 1, 2, 4 and 8 are all the factors (also called divisors) of 8.

A **multiple** of an integer is formed by multiplying that integer by any integer, so 8, 16, 24, and 32 are some of the multiples of 8. Additionally, negative multiples are possible (−8, −16, −24, −32, etc.), but the GMAT does not test negative multiples directly. Also, zero (0) is technically a multiple of every number, because that number times zero (an integer) equals zero.

Note that an integer is always both a factor and a multiple of itself, and that 1 is a factor of **every** integer.

An easy way to find all the factors of a SMALL number is to use **factor pairs**. Factor pairs for any integer are the pairs of factors that, when multiplied together, yield that integer.

To find the factor pairs of a number such as 72, you should start with the automatic factors: 1 and 72 (the number itself). Then, simply “walk upwards” from 1, testing to see whether different numbers are factors of 72. Once you find a number that is a factor of 72, find its partner by dividing 72 by the factor. Keep walking upwards until all factors are exhausted.

### Step by step:

- (1) Make a table with 2 columns labeled “Small” and “Large.”
- (2) Start with 1 in the small column and 72 in the large column.
- (3) Test the next possible factor of 72 (which is 2). 2 is a factor of 72, so write “2” underneath the “1” in your table. Divide 72 by 2 to find the factor pair:  $72 \div 2 = 36$ . Write “36” in the large column.
- (4) Test the next possible factor of 72 (which is 3). Repeat this process until the numbers in the small and the large columns run into each other. In this case, once we have tested 8 and found that 9 was its paired factor, we can stop.

Small	Large
1	72
2	36
3	24
4	18
6	12
8	9

## Fewer Factors, More Multiples

Sometimes it is easy to confuse factors and multiples. The mnemonic “Fewer Factors, More Multiples” should help you remember the difference. Factors divide into an integer and are therefore less than or equal to that integer. Positive multiples, on the other hand, multiply out from an integer and are therefore greater than or equal to that integer.

Any integer only has a limited number of factors. For example, there are only four factors of 8: 1, 2, 4, and 8. By contrast, there is an infinite number of multiples of an integer. For example, the first 5 positive multiples of 8 are 8, 16, 24, 32, and 40, but you could go on listing multiples of 8 forever.

Factors, multiples, and divisibility are very closely related concepts. For example, 3 is a factor of 12. This is the same as saying that 12 is a multiple of 3, or that 12 is divisible by 3.

You can use factor pairs to determine all of the factors of any integer, in theory, but the process works best with small numbers.

On the GMAT, this terminology is often used interchangeably in order to make the problem seem harder than it actually is. Be aware of the different ways that the GMAT can phrase information about divisibility. Moreover, try to convert all such statements to the same terminology. For example, **all** of the following statements **say exactly the same thing**:

- 12 is divisible by 3
- 12 is a multiple of 3
- $\frac{12}{3}$  is an integer
- $12 = 3n$ , where  $n$  is an integer
- 12 items can be shared among 3 people so that each person has the same number of items.
- 3 is a divisor of 12, or 3 is a factor of 12
- 3 divides 12
- $\frac{12}{3}$  yields a remainder of 0
- 3 “goes into” 12 evenly

The GMAT can state that  $x$  is divisible by  $y$  in several different ways—learn these different phrasings and mentally convert them to a single form when you see them!

## Divisibility and Addition/Subtraction

If you add two multiples of 7, you get another multiple of 7. Try it:  $35 + 21 = 56$ . This should make sense:  $(5 \times 7) + (3 \times 7) = (5 + 3) \times 7 = 8 \times 7$ .

Likewise, if you subtract two multiples of 7, you get another multiple of 7. Try it:  $35 - 21 = 14$ . Again, we can see why:  $(5 \times 7) - (3 \times 7) = (5 - 3) \times 7 = 2 \times 7$ .

This pattern holds true for the multiples of any integer  $N$ . **If you add or subtract multiples of  $N$ , the result is a multiple of  $N$ .** You can restate this principle using any of the disguises above: for instance, if  $N$  is a divisor of  $x$  and of  $y$ , then  $N$  is a divisor of  $x + y$ .

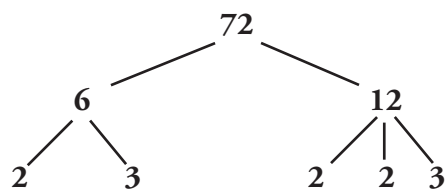
## Primes

Prime numbers are a very important topic on the GMAT. A prime number is any positive integer larger than 1 with exactly two factors: 1 and itself. In other words, a prime number has NO factors other than 1 and itself. For example, 7 is prime because the only factors of 7 are 1 and 7. However, 8 is not prime because it is divisible by 2 and 4.

Note that the number 1 is not considered prime, as it has only one factor (itself). **Thus, the first prime number is 2, which is also the only even prime.** The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. You should memorize these primes.

## Prime Factorization

One very helpful way to analyze a number is to break it down into its prime factors. This can be done by creating a prime factor tree, as shown to the right with the number 72. Simply test different numbers to see which ones “go into” 72 without leaving a remainder. Once you find such a number, then split 72 into factors. For example, 72 is divisible by 6, so it can be split into 6 and  $72 \div 6$ , or 12. Then repeat this process on the factors of 72 until every branch on the tree ends at a prime number. Once we only have primes, we stop, because we cannot split prime numbers into two smaller factors. In this example, 72 splits into 5 total prime factors (including repeats):  $2 \times 3 \times 2 \times 2 \times 3$ .



Prime factorization is an extremely important tool to use on the GMAT. One reason is that once we know the prime factors of a number, we can determine ALL the factors of that number, even large numbers. The factors can be found by building all the possible products of the prime factors.

On the GMAT, prime factorization is useful for many other applications in addition to enumerating factors. Some other situations in which you might need to use prime factorization include the following:

- (1) Determining whether one number is divisible by another number
- (2) Determining the greatest common factor of two numbers
- (3) Reducing fractions
- (4) Finding the least common multiple of two (or more) numbers
- (5) Simplifying square roots
- (6) Determining the exponent on one side of an equation with integer constraints

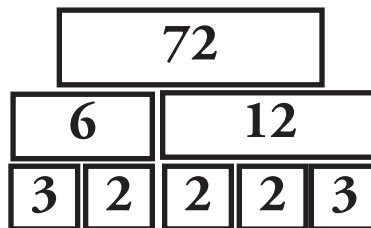
Prime numbers are the building blocks of integers. Many problems require variables to be integers, and you can often solve or simplify these problems by analyzing primes. A simple rule to remember is this: **if the problem states or assumes that a number is an integer, you MAY need to use prime factorization to solve the problem.**

### Factor Foundation Rule

The GMAT expects you to know the factor foundation rule: **if  $a$  is a factor of  $b$ , and  $b$  is a factor of  $c$ , then  $a$  is a factor of  $c$ .** In other words, any integer is divisible by all of its factors—and it is also divisible by all of the FACTORS of its factors.

For example, if 72 is divisible by 12, then 72 is also divisible by all the factors of 12 (1, 2, 3, 4, 6, and 12). Written another way, if 12 is a factor of 72, then all the factors of 12 are also factors of 72. The Factor Foundation Rule allows us to conceive of factors as building blocks in a foundation. 12 and 6 are factors, or building blocks, of 72 (because  $12 \times 6$  builds 72).

The number 12, in turn, is built from its own factors; for example,  $4 \times 3$  builds 12. Thus, if 12 is part of the foundation of 72 and 12 in turn rests on the foundation built by its prime factors (2, 2, and 3), then 72 is also built on the foundation of 2, 2, and 3.



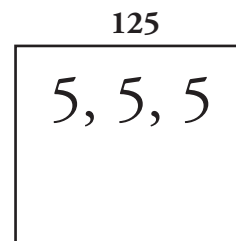
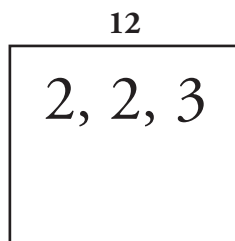
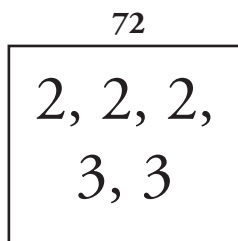
Going further, we can build almost any factor of 72 out of the bottom level of the foundation. For instance, we can see that 8 is a factor of 72, because we can build 8 out of the three 2's in the bottom row ( $8 = 2 \times 2 \times 2$ ).

We say **almost** any factor, because one of the factors cannot be built out of the building blocks in the foundation: the number 1. Remember that the number 1 is not prime, but it is still a factor of every integer. Except for the number 1, every factor of 72 can be built out of the lowest level of 72 building blocks.

Think of the prime factors of an integer as that integer's "foundation," from which all factors of that number (except 1) can be built.

## The Prime Box

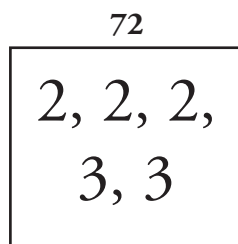
The easiest way to work with the Factor Foundation Rule is with a tool called a Prime Box. A Prime Box is exactly what its name implies: a box that holds all the prime factors of a number (in other words, the lowest-level building blocks). Here are prime boxes for 72, 12, and 125:



Every integer larger than 1 has a unique prime factorization.

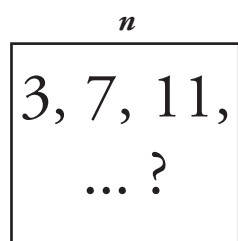
Notice that we must repeat copies of the prime factors if the number has multiple copies of that prime factor. You can use the prime box to test whether or not a specific number is a factor of another number.

Is 27 a factor of 72?



$27 = 3 \times 3 \times 3$ . But we can see that 72 only has two 3's in its prime box. Therefore we cannot make 27 from the prime factors of 72. Thus, 27 is not a factor of 72.

Given that the integer  $n$  is divisible by 3, 7, and 11, what other numbers must be divisors of  $n$ ?



Since we know that 3, 7, and 11 are prime factors of  $n$ , we know that  $n$  must also be divisible by all the possible **products** of the primes in the box: 21, 33, 77, and 231.

Without even knowing what  $n$  is, we have found 4 more of its factors: 21, 33, 77, and 231.

Notice also the ellipses and question mark (“... ?”) in the prime box of  $n$ . This reminds us that we have created a **partial prime box** of  $n$ . Whereas the COMPLETE set of prime factors of 72 can be calculated and put into its prime box, we only have a PARTIAL list of prime factors of  $n$ , because  $n$  is an unknown number. We know that  $n$  is divisible by 3, 7, and 11, but we do NOT know what additional primes, if any,  $n$  has in its prime box.

Most of the time, when building a prime box for a VARIABLE, we will use a partial prime box, but when building a prime box for a NUMBER, we will use a complete prime box.

## Greatest Common Factor and Least Common Multiple

Frequently on the GMAT, you may have to find the Greatest Common Factor (GCF) or Least Common Multiple (LCM) of a set of two or more numbers.

**Greatest Common Factor (GCF):** the largest divisor of two or more integers.

**Least Common Multiple (LCM):** the smallest multiple of two or more integers.

It is likely that you already know how to find both the GCF and the LCM. For example,

when you reduce the fraction  $\frac{9}{12}$  to  $\frac{3}{4}$ , you are dividing both the numerator (9) and denominator (12) by 3, which is the GCF of 9 and 12. When you add together the frac-

tions  $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$ , you convert the fractions to thirtieths:  $\frac{1}{2} + \frac{1}{3} + \frac{1}{5} = \frac{15}{30} + \frac{10}{30} + \frac{6}{30} = \frac{31}{30}$ .

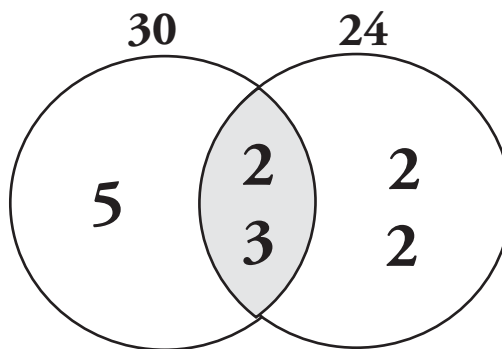
Why thirtieths? The reason is that 30 is the LCM of the denominators: 2, 3, and 5.

The GCF and the LCM can best be understood visually by using a Venn diagram.

### FINDING GCF AND LCM USING VENN DIAGRAMS

One way that you can visualize the GCF and LCM of two numbers is by placing prime factors into a **Venn diagram**—a diagram of circles showing the overlapping and non-overlapping elements of two sets. To find the GCF and LCM of two numbers using a Venn diagram, perform the following steps:

- (1) Factor the numbers into primes.
- (2) Create a Venn diagram.
- (3) Place each common factor, including copies of common factors appearing more than once, into the shared area of the diagram (the shaded region to the right).
- (4) Place the remaining (non-common) factors into the non-shared areas.

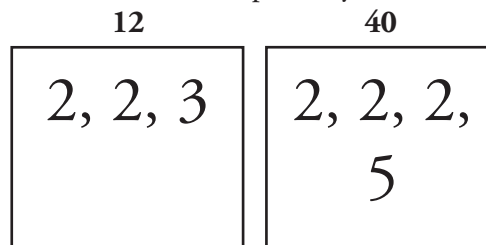


The Venn diagram above illustrates how to determine the GCF and LCM of 30 and 24. **The GCF is the product of primes in the overlapping region:**  $2 \times 3 = 6$ . **The LCM is the product of ALL primes in the diagram:**  $5 \times 2 \times 3 \times 2 \times 2 = 120$ .

Compute the GCF and LCM of 12 and 40 using the Venn diagram approach.

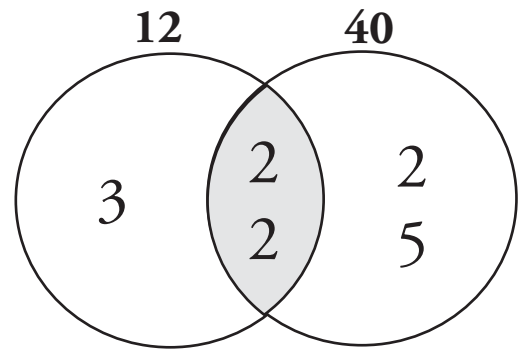
The prime factorizations of 12 and 40 are  $2 \times 2 \times 3$  and  $2 \times 2 \times 2 \times 5$ , respectively:

The only common factors of 12 and 40 are two 2's. Therefore, we place two 2's in the shared area of the Venn diagram (on the next page) and remove them from BOTH prime factorizations. Then, place the remaining factors in the zones belonging exclusively to 12 and 40. These two outer regions **must have no primes in common!**



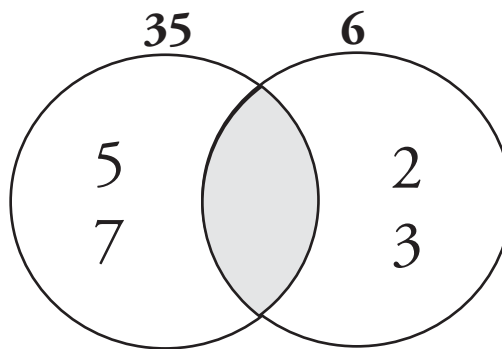
The GCF of 12 and 40 is therefore  $2 \times 2 = 4$ , the product of the primes in the **shared area**. (An easy way to remember this is that the “common factors” are in the “common area.”)

The LCM is  $2 \times 2 \times 2 \times 3 \times 5 = 120$ , the product of **all** the primes in the diagram.



The product of the shared primes is the GCF. The product of all the primes (counting shared primes just once) is the LCM.

Note that if two numbers have NO primes in common, then their GCF is 1 and their LCM is simply their product. For example, 35 ( $= 5 \times 7$ ) and 6 ( $= 2 \times 3$ ) have no prime numbers in common. Therefore, their GCF is 1 (the common factor of *all* positive integers) and their LCM is  $35 \times 6 = 210$ . Be careful: even though you have no primes in the common area, the GCF is not 0 but 1.



## Remainders

The number 17 is not divisible by 5. When you divide 17 by 5, using long division, you get a **remainder**: a number left over. In this case, the remainder is 2.

$$\begin{array}{r} 3 \\ 5 \overline{) 17} \\ \underline{-15} \\ 2 \end{array}$$

We can also write that 17 is 2 more than 15, or 2 more than a multiple of 5. In other words, we can write  $17 = 15 + 2 = 3 \times 5 + 2$ . Every number that leaves a remainder of 2 after it is divided by 5 can be written this way: as a multiple of 5, plus 2.

On simpler remainder problems, it is often easiest to pick numbers. Simply add the desired remainder to a multiple of the divisor. For instance, if you need a number that leaves a remainder of 4 after division by 7, first pick a multiple of 7: say, 14. Then add 4 to get 18, which satisfies the requirement ( $18 = 7 \times 2 + 4$ ).

## Problem Set

For problems #1–12, use one or more prime boxes, if appropriate, to answer each question: YES, NO, or CANNOT BE DETERMINED. If your answer is CANNOT BE DETERMINED, use two numerical examples to show how the problem could go either way. All variables in problems #1 through #12 are assumed to be integers unless otherwise indicated.

1. If  $a$  is divided by 7 or by 18, an integer results. Is  $\frac{a}{42}$  an integer?
2. If 80 is a factor of  $r$ , is 15 a factor of  $r$ ?
3. Given that 7 is a factor of  $n$  and 7 is a factor of  $p$ , is  $n + p$  divisible by 7?
4. Given that 8 is not a factor of  $g$ , is 8 a factor of  $2g$ ?
5. If  $j$  is divisible by 12 and 10, is  $j$  divisible by 24?
6. If 12 is a factor of  $xyz$ , is 12 a factor of  $xy$ ?
7. Given that 6 is a divisor of  $r$  and  $r$  is a factor of  $s$ , is 6 a factor of  $s$ ?
8. If 24 is a factor of  $h$  and 28 is a factor of  $k$ , must 21 be a factor of  $hk$ ?
9. If 6 is not a factor of  $d$ , is  $12d$  divisible by 6?
10. If  $k$  is divisible by 6 and  $3k$  is not divisible by 5, is  $k$  divisible by 10?
11. If 60 is a factor of  $u$ , is 18 a factor of  $u$ ?
12. If  $s$  is a multiple of 12 and  $t$  is a multiple of 12, is  $7s + 5t$  a multiple of 12?

Solve Problems #13–15:

13. What is the greatest common factor of 420 and 660?
14. What is the least common multiple of 18 and 24?
15. A skeet shooting competition awards prizes as follows: the first place winner receives 11 points, the second place winner receives 7 points, the third place finisher receives 5 points, and the fourth place finisher receives 2 points. No other prizes are awarded. John competes in the skeet shooting competition several times and receives points every time he competes. If the product of all of the points he receives equals 84,700, how many times does he participate in the competition?

1. YES:

$a$

$2, 3, 3,$   
 $7, \dots ?$

If  $a$  is divisible by 7 and by 18, its prime factors include 2, 3, 3, and 7, as indicated by the prime box to the left. Therefore, any integer that can be constructed as a product of any of these prime factors is also a factor of  $a$ .  $42 = 2 \times 3 \times 7$ . Therefore, 42 is also a factor of  $a$ .

2. CANNOT BE DETERMINED:

$r$

$2, 2, 2,$   
 $2, 5, \dots ?$

If  $r$  is divisible by 80, its prime factors include 2, 2, 2, 2, and 5, as indicated by the prime box to the left. Therefore, any integer that can be constructed as a product of any of these prime factors is also a factor of  $r$ .  $15 = 3 \times 5$ . Since the prime factor 3 is not in the prime box, we cannot determine whether 15 is a factor of  $r$ . As numerical examples, we could take  $r = 80$ , in which case 15 is NOT a factor of  $r$ , or  $r = 240$ , in which case 15 IS a factor of  $r$ .

3. YES: If 2 numbers are both multiples of the same number, then their SUM is also a multiple of that same number. Since  $n$  and  $p$  share the common factor 7, the sum of  $n$  and  $p$  must also be divisible by 7.

4. CANNOT BE DETERMINED:

$2g$

$2,$	$g$
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <math>(\text{not } 2,</math>  <math>2, 2)</math> </div>	

In order for 8 to be a factor of  $2g$ , we would need two more 2's in the prime box. By the Factor Foundation Rule,  $g$  would need to be divisible by 4. We know that  $g$  is not divisible by 8, but there are certainly integers that are divisible by 4 and not by 8, such as 4, 12, 20, 28, etc. However, while we cannot conclude that  $g$  is **not** divisible by 4, we cannot be certain that  $g$  **is** divisible by 4, either. As numerical examples, we could take  $g = 5$ , in which case 8 is NOT a factor of  $2g$ , or  $g = 4$ , in which case 8 IS a factor of  $2g$ .

5. CANNOT BE DETERMINED:

$j$

 $2, 2, 3,$   
 $\dots ?$

$j$

 $2, 5,$   
 $\dots ?$

$\rightarrow$

$j$

 $2, 2, 3,$   
 $5, \dots ?$

If  $j$  is divisible by 12 and by 10, its prime factors include 2, 2, 3, and 5, as indicated by the prime box to the left. There are only TWO 2's that are definitely in the prime factorization of  $j$ , because the 2 in the prime factorization of 10

may be REDUNDANT—that is, it may be the SAME 2 as one of the 2's in the prime factorization of 12.

$24 = 2 \times 2 \times 2 \times 3$ . There are only two 2's in the prime box of  $j$ ; 24 requires three 2's. Therefore, 24 is not necessarily a factor of  $j$ .

As another way to prove that we cannot determine whether 24 is a factor of  $j$ , consider 60. The number 60 is divisible by both 12 and 10. However, it is NOT divisible by 24. Therefore,  $j$  could equal 60, in which case it is not divisible by 24. Alternatively,  $j$  could equal 120, in which case it IS divisible by 24.