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GEOMETRY

Math Strategy Guide

This comprehensive guide illustrates every geometric principle, formula, and problem type tested on the GMAT. Understand and master the intricacies of shapes, planes, lines, angles, and objects.

Geometry GMAT Strategy Guide, Fourth Edition

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May 1st, 2009

Thank you for picking up one of the Manhattan GMAT Strategy Guides—we hope that it refreshes your memory of the junior-high math that you haven't used in years. Maybe it will even teach you a new thing or two.

As with most accomplishments, there were many people involved in the various iterations of the book that you're holding. First and foremost is Zeke Vanderhoek, the founder of Manhattan GMAT. Zeke was a lone tutor in New York when he started the Company in 2000. Now, nine years later, MGMAT has Instructors and offices nationwide, and the Company contributes to the studies and successes of thousands of students each year.

Our 4th Edition Strategy Guides are based on the continuing experiences of our Instructors and our students. We owe much of these latest editions to the insight provided by our students. On the Company side, we are indebted to many of our Instructors, including but not limited to Josh Braslow, Dan Gonzalez, Mike Kim, Stacey Koprince, Ben Ku, Jadran Lee, David Mahler, Ron Purewal, Tate Shafer, Emily Sledge, and of course Chris Ryan, the Company's Lead Instructor and Director of Curriculum Development.

At Manhattan GMAT, we continually aspire to provide the best Instructors and resources possible. We hope that you'll find our dedication manifest in this book. If you have any comments or questions, please e-mail me at andrew.yang@manhattangmat.com. I'll be sure that your comments reach Chris and the rest of the team—and I'll read them too.

Best of luck in preparing for the GMAT!

Sincerely,

Andrew Yang
Chief Executive Officer
ManhattanGMAT

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PART I: GENERAL

This part of the book covers both basic and intermediate topics within *Geometry*. Complete Part I before moving on to Part II: Advanced.

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Chapter 1

of

GEOMETRY

POLYGONS

In This Chapter . . .

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- Quadrilaterals: An Overview
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- Polygons and Area
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POLYGONS

A polygon is defined as a closed shape formed by line segments. The polygons tested on the GMAT include the following:

- Three-sided shapes (Triangles)
- Four-sided shapes (Quadrilaterals)
- Other polygons with n sides (where n is five or more)

This section will focus on polygons of four or more sides. In particular, the GMAT emphasizes quadrilaterals—or four-sided polygons—including trapezoids, parallelograms, and special parallelograms, such as rhombuses, rectangles, and squares.

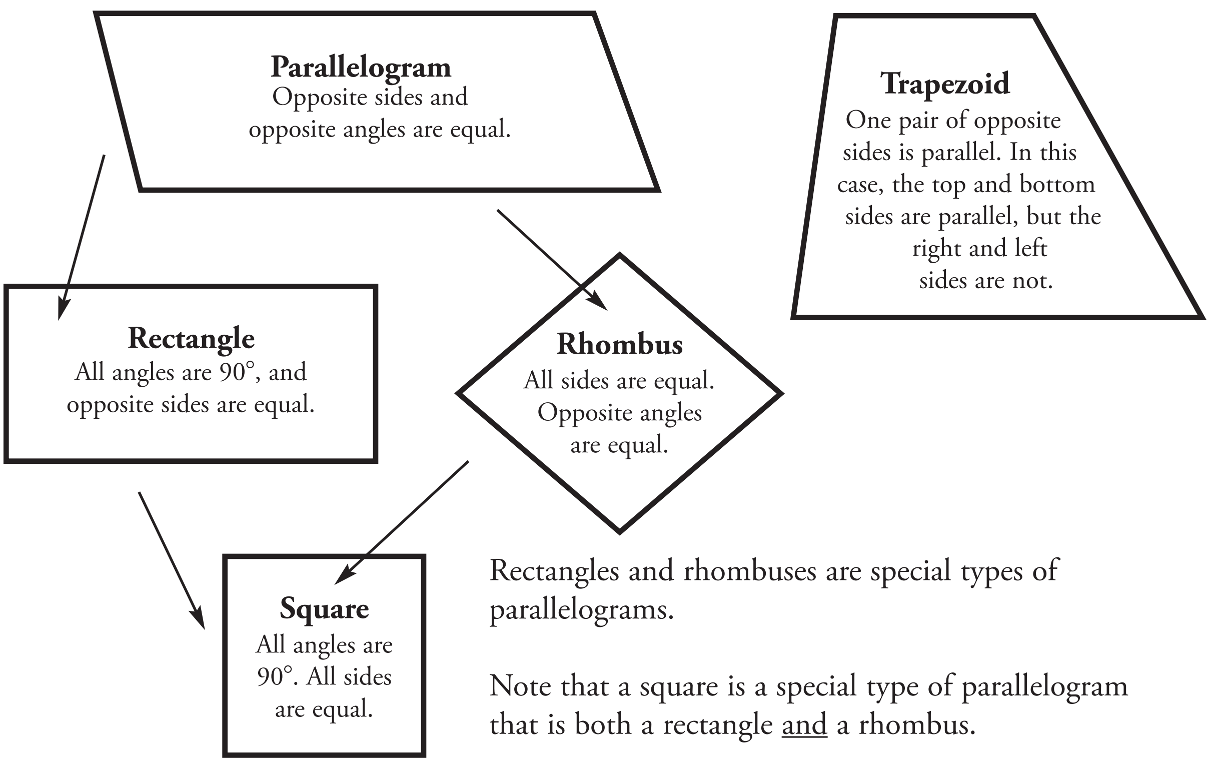
Polygons are two-dimensional shapes—they lie in a plane. The GMAT tests your ability to work with different measurements associated with polygons. The measurements you must be adept with are (1) interior angles, (2) perimeter, and (3) area.

A polygon is a closed shape formed by line segments.

The GMAT also tests your knowledge of three-dimensional shapes formed from polygons, particularly rectangular solids and cubes. The measurements you must be adept with are (1) surface area and (2) volume.

Quadrilaterals: An Overview

The most common polygon tested on the GMAT, aside from the triangle, is the quadrilateral (any four-sided polygon). Almost all GMAT polygon problems involve the special types of quadrilaterals shown below.



Rectangles and rhombuses are special types of parallelograms.

Note that a square is a special type of parallelogram that is both a rectangle <u>and</u> a rhombus.

Polygons and Interior Angles

The sum of the interior angles of a given polygon depends only on the **number of sides in the polygon**. The following chart displays the relationship between the type of polygon and the sum of its interior angles.

The sum of the interior angles of a polygon follows a specific pattern that depends on n, the number of sides that the polygon has. This sum is always 180° times 2 less than n (the number of sides), because the polygon can be cut into (n – 2) triangles, each of which contains 180°.

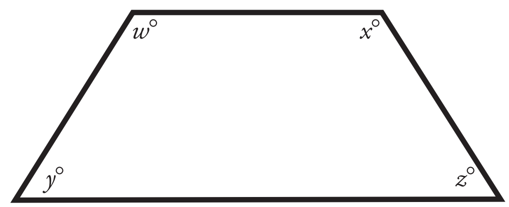
Polygon	# of Sides	Sum of Interior Angles
Triangle	3	180°
Quadrilateral	4	360°
Pentagon	5	540°
Hexagon	6	720°

Another way to find the sum of the interior angles in a polygon is to divide the polygon into triangles. The interior angles of each triangle sum to 180°.

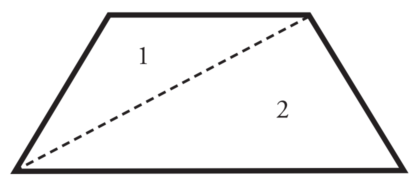
This pattern can be expressed with the following formula:

$$\boxed{(n - 2) \times 180 = \text{Sum of Interior Angles of a Polygon}}$$

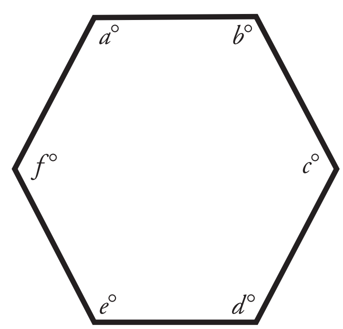
Since this polygon has four sides, the sum of its interior angles is (4 – 2)180 = 2(180) = 360°.



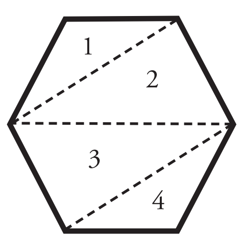
Alternatively, note that a quadrilateral can be cut into two triangles by a line connecting opposite corners. Thus, the sum of the angles = 2(180) = 360°.



Since the next polygon has six sides, the sum of its interior angles is (6 – 2)180 = 4(180) = 720°.



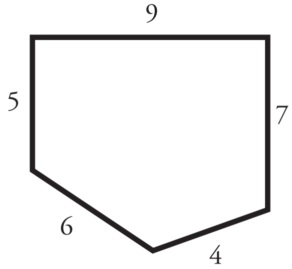
Alternatively, note that a hexagon can be cut into four triangles by three lines connecting corners.



Thus, the sum of the angles = 4(180) = 720°.

By the way, the corners of polygons are also known as vertices (singular: vertex).

Polygons and Perimeter



The perimeter refers to the distance around a polygon, or the sum of the lengths of all the sides. The amount of fencing needed to surround a yard would be equivalent to the perimeter of that yard (the sum of all the sides).

The perimeter of the pentagon to the left is:
$9 + 7 + 4 + 6 + 5 = \mathbf{31}$.

Polygons and Area

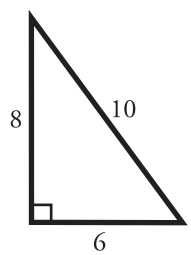
The area of a polygon refers to the space inside the polygon. Area is measured in square units, such as cm^2 (square centimeters), m^2 (square meters), or ft^2 (square feet). For example, the amount of space that a garden occupies is the area of that garden.

You must memorize the formulas for the area of a triangle and for the area of the quadrilaterals shown in this section.

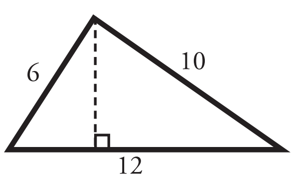
On the GMAT, there are two polygon area formulas you MUST know:

1) Area of a TRIANGLE $= \dfrac{\textbf{Base} \times \textbf{Height}}{\textbf{2}}$

The base refers to the bottom side of the triangle. The height ALWAYS refers to a line that is perpendicular (at a 90° angle) to the base.

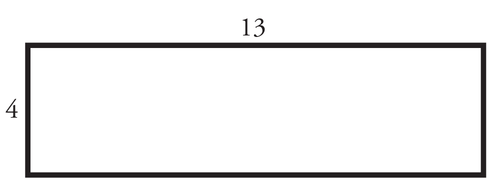


In this triangle, the base is 6 and the height (perpendicular to the base) is 8. The area = $(6 \times 8) \div 2 = 48 \div 2 = 24$.



In this triangle, the base is 12, but the height is not shown. Neither of the other two sides of the triangle is perpendicular to the base. In order to find the area of this triangle, we would first need to determine the height, which is represented by the dotted line.

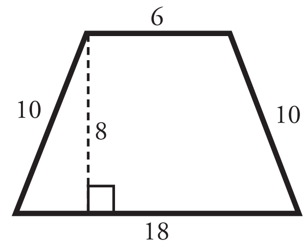
2) Area of a RECTANGLE = **Length** × **Width**



The length of this rectangle is 13, and the width is 4. Therefore, the area = $13 \times 4 = 52$.

The GMAT will occasionally ask you to find the area of a polygon more complex than a simple triangle or rectangle. The following formulas can be used to find the areas of other types of quadrilaterals:

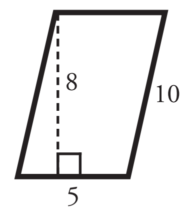
3) Area of a TRAPEZOID = $\dfrac{\textbf{(Base}_1 + \textbf{Base}_2\textbf{)} \times \textbf{Height}}{\textbf{2}}$



Note that the height refers to a line perpendicular to the two bases, which are parallel. (You often have to draw in the height, as in this case.) In the trapezoid shown, $\text{base}_1 = 18$, $\text{base}_2 = 6$, and the height = 8. The area = $(18 + 6) \times 8 \div 2 = 96$. Another way to think about this is to take the *average* of the two bases and multiply it by the height.

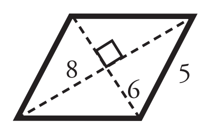
Notice that most of these formulas involve finding a base and a line perpendicular to that base (a height).

4) Area of any PARALLELOGRAM = **Base** × **Height**



Note that the height refers to the line perpendicular to the base. (As with the trapezoid, you often have to draw in the height.) In the parallelogram shown, the base = 5 and the height = 8. Therefore, the area is $5 \times 8 = 40$.

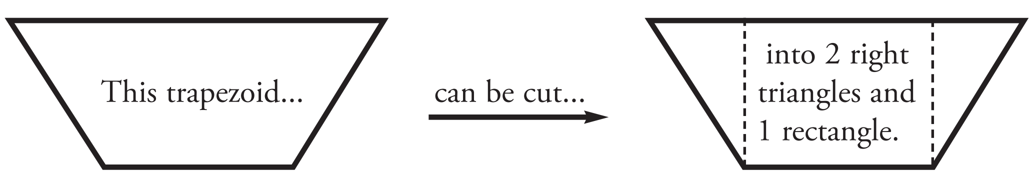
5) Area of a RHOMBUS = $\dfrac{\textbf{Diagonal}_1 \times \textbf{Diagonal}_2}{\textbf{2}}$



Note that the diagonals of a rhombus are ALWAYS perpendicular bisectors (meaning that they cut each other in half at a 90° angle).

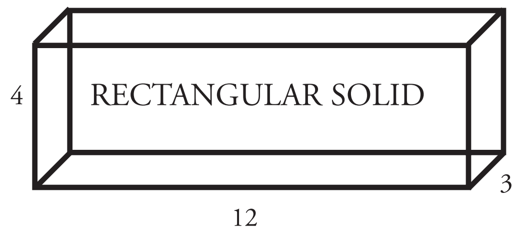
The area of this rhombus is $\dfrac{6 \times 8}{2} = \dfrac{48}{2} = 24$.

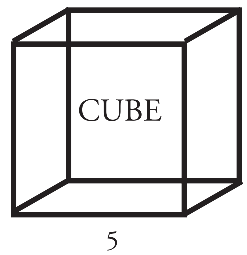
Although these formulas are very useful to memorize for the GMAT, you may notice that all of the above shapes can actually be divided into some combination of rectangles and right triangles. Therefore, if you forget the area formula for a particular shape, simply cut the shape into rectangles and right triangles, and then find the areas of these individual pieces. For example:



3 Dimensions: Surface Area

The GMAT tests two particular three-dimensional shapes formed from polygons: the rectangular solid and the cube. Note that a cube is just a special type of rectangular solid.





You do not need to memorize a formula for surface area. Simply find the sum of all of the faces.

The surface area of a three-dimensional shape is the amount of space on the surface of that particular object. For example, the amount of paint that it would take to fully cover a rectangular box could be determined by finding the surface area of that box. As with simple area, surface area is measured in square units such as inches^2 (square inches) or ft^2 (square feet).

Surface Area = the SUM of the areas of ALL of the faces

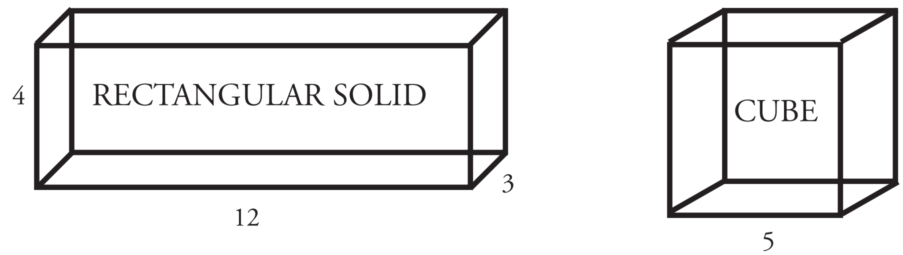
Both a rectangular solid and a cube have **six faces**.

To determine the surface area of a rectangular solid, you must find the area of each face. Notice, however, that in a rectangular solid, the front and back faces have the same area, the top and bottom faces have the same area, and the two side faces have the same area. In the solid above, the area of the front face is equal to $12 \times 4 = 48$. Thus, the back face also has an area of 48. The area of the bottom face is equal to $12 \times 3 = 36$. Thus, the top face also has an area of 36. Finally, each side face has an area of $3 \times 4 = 12$. Therefore, the surface area, or the sum of the areas of all six faces equals $48(2) + 36(2) + 12(2) = 192$.

To determine the surface area of a cube, you only need the length of one side. We can see from the cube above that a cube is made of six square surfaces. First, find the area of one face: $5 \times 5 = 25$. Then, multiply by six to account for all of the faces: $6 \times 25 = 150$.

3 Dimensions: Volume

The volume of a three-dimensional shape is the amount of "stuff" it can hold. For example, the amount of liquid that a rectangular milk carton holds can be determined by finding the volume of the carton. Volume is measured in cubic units such as $inches^3$ (cubic inches) or ft^3 (cubic feet).



Volume = Length × Width × Height

Another way to think about this formula is that the volume is equal to the area of the base multiplied by the height.

By looking at the rectangular solid above, we can see that the length is 12, the width is 3, and the height is 4. Therefore, the volume is $12 \times 3 \times 4 = 144$.

In a cube, all three of the dimensions—length, width, and height—are identical. Therefore, knowing the measurement of just one side of the cube is sufficient to find the volume. In the cube above, the volume is $5 \times 5 \times 5 = 125$.

Beware of a GMAT volume trick:

> How many books, each with a volume of 100 in^3, can be packed into a crate with a volume of 5,000 in^3?

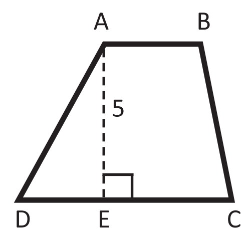
It is tempting to answer "50 books" (since $50 \times 100 = 5{,}000$). However, this is incorrect, because we do not know the exact dimensions of each book! One book might be $5 \times 5 \times 4$, while another book might be $20 \times 5 \times 1$. Even though both have a volume of 100 in^3, they have different rectangular shapes. Without knowing the exact shapes of all the books, we cannot tell whether they would all fit into the crate. Remember, when you are fitting 3-dimensional objects into other 3-dimensional objects, knowing the respective volumes is not enough. We must know the specific dimensions (length, width, and height) of each object to determine whether the objects can fit without leaving gaps.

Problem Set (Note: Figures are not drawn to scale.)

1. Frank the Fencemaker needs to fence in a rectangular yard. He fences in the entire yard, except for one 40-foot side of the yard. The yard has an area of 280 square feet. How many feet of fence does Frank use?

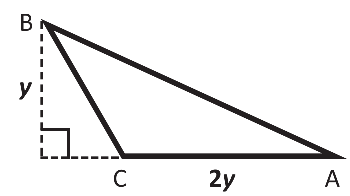
2. A pentagon has three sides with length x, and two sides with the length $3x$. If x is $\frac{2}{3}$ of an inch, what is the perimeter of the pentagon?

3. ABCD is a quadrilateral, with AB parallel to CD (see figure). E is a point between C and D such that AE represents the height of ABCD, and E is the midpoint of CD. If AB is 4 inches long, AE is 5 inches long, and the area of triangle AED is 12.5 square inches, what is the area of ABCD?



4. A rectangular tank needs to be coated with insulation. The tank has dimensions of 4 feet, 5 feet, and 2.5 feet. Each square foot of insulation costs $20. How much will it cost to cover the surface of the tank with insulation?

5. Triangle ABC (see figure) has a base of $2y$, a height of y, and an area of 49. What is y?



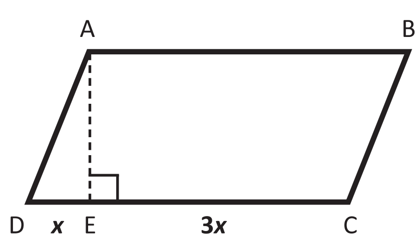
6. 40 percent of Andrea's living room floor is covered by a carpet that is 4 feet by 9 feet. What is the area of her living room floor?

7. If the perimeter of a rectangular flower bed is 30 feet, and its area is 44 square feet, what is the length of each of its shorter sides?

8. There is a rectangular parking lot with a length of $2x$ and a width of x. What is the ratio of the perimeter of the parking lot to the area of the parking lot, in terms of x?

9. A rectangular solid has a square base, with each side of the base measuring 4 meters. If the volume of the solid is 112 cubic meters, what is the surface area of the solid?

10. ABCD is a parallelogram (see figure). The ratio of DE to EC is 1 : 3. AE has a length of 3. If quadrilateral ABCE has an area of 21, what is the area of ABCD?



11. A swimming pool has a length of 30 meters, a width of 10 meters, and an average depth of 2 meters. If a hose can fill the pool at a rate of 0.5 cubic meters per minute, how many hours will it take the hose to fill the pool?

1. **54 feet:** We know that one side of the yard is 40 feet long; let us call this the length. We also know that the area of the yard is 280 square feet. In order to determine the perimeter, we must know the width of the yard.

$$A = l \times w$$
$$280 = 40w$$
$$w = 280 \div 40 = 7 \text{ feet}$$

Frank fences in the two 7-foot sides and one of the 40-foot sides. 40 + 2(7) = 54.

2. **6 inches:** The perimeter of a pentagon is the sum of its five sides: $x + x + x + 3x + 3x = 9x$. If x is 2/3 of an inch, the perimeter is 9(2/3), or 6 inches.

3. **35 in^2:** If E is the midpoint of C, then CE = DE = x. We can determine the length of x by using what we know about the area of triangle AED.

$$A = \frac{b \times h}{2} \qquad 12.5 = \frac{5x}{2}$$

$$25 = 5x$$
$$x = 5$$

Therefore, the length of CD is $2x$, or 10.

To find the area of the trapezoid, use the formula: $A = \frac{b_1 + b_2}{2} \times h$

$$= \frac{4 + 10}{2} \times 5$$

$$= 35 \text{ in}^2$$

4. **$1,700:** To find the surface area of a rectangular solid, sum the individual areas of all six faces:

Top and Bottom:	$5 \times 4 = 20$	$\rightarrow$	$2 \times 20 = 40$
Side 1:	$5 \times 2.5 = 12.5$	$\rightarrow$	$2 \times 12.5 = 25$
Side 2:	$4 \times 2.5 = 10$	$\rightarrow$	$2 \times 10 = 20$

$$40 + 25 + 20 = 85 \text{ ft}^2$$

Covering the entire tank will cost 85 × $20 = $1,700.

5. **7:** The area of a triangle is equal to half the base times the height. Therefore, we can write the following relationship:

$$\frac{2y(y)}{2} = 49$$

$$y^2 = 49$$
$$y = 7$$