FRACTIONS, DECIMALS, & PERCENTS

Math Strategy Guide

This guide provides an in-depth look at the variety of GMAT questions that test your knowledge of fractions, decimals, and percents. Learn to see the connections among these part–whole relationships and practice implementing strategic shortcuts.

Fractions, Decimals, and Percents GMAT Strategy Guide, Fourth Edition

10–digit International Standard Book Number: 0-9824238-2-9
13–digit International Standard Book Number: 978-0-9824238-2-0

8 GUIDE INSTRUCTIONAL SERIES

Math GMAT Strategy Guides

Number Properties (ISBN: 978-0-9824238-4-4)

Fractions, Decimals, & Percents (ISBN: 978-0-9824238-2-0)

Equations, Inequalities, & VICs (ISBN: 978-0-9824238-1-3)

Word Translations (ISBN: 978-0-9824238-7-5)

Geometry (ISBN: 978-0-9824238-3-7)

Verbal GMAT Strategy Guides

Critical Reasoning (ISBN: 978-0-9824238-0-6)

Reading Comprehension (ISBN: 978-0-9824238-5-1)

Sentence Correction (ISBN: 978-0-9824238-6-8)

*Manhattan*GMAT
the new standard

May 1st, 2009

Dear Student,

Thank you for picking up one of the Manhattan GMAT Strategy Guides—we hope that it refreshes your memory of the junior-high math that you haven't used in years. Maybe it will even teach you a new thing or two.

As with most accomplishments, there were many people involved in the various iterations of the book that you're holding. First and foremost is Zeke Vanderhoek, the founder of Manhattan GMAT. Zeke was a lone tutor in New York when he started the Company in 2000. Now, nine years later, MGMAT has Instructors and offices nationwide, and the Company contributes to the studies and successes of thousands of students each year.

Our 4th Edition Strategy Guides are based on the continuing experiences of our Instructors and our students. We owe much of these latest editions to the insight provided by our students. On the Company side, we are indebted to many of our Instructors, including but not limited to Josh Braslow, Dan Gonzalez, Mike Kim, Stacey Koprince, Ben Ku, Jadran Lee, David Mahler, Ron Purewal, Tate Shafer, Emily Sledge, and of course Chris Ryan, the Company's Lead Instructor and Director of Curriculum Development.

At Manhattan GMAT, we continually aspire to provide the best Instructors and resources possible. We hope that you'll find our dedication manifest in this book. If you have any comments or questions, please e-mail me at andrew.yang@manhattangmat.com. I'll be sure that your comments reach Chris and the rest of the team—and I'll read them too.

Best of luck in preparing for the GMAT!

Sincerely,

Andrew Yang
Chief Executive Officer
Manhattan GMAT

www.manhattangmat.com 138 West 25th St., 9th Floor NY, NY 10001 Tel: 212-721-7400 Fax: 646-514-7425

TABLE OF CONTENTS

Part I: General

Part II: Advanced

g

PART I: GENERAL

This part of the book covers both basic and intermediate topics within *Fractions, Decimals, & Percents*. Complete Part I before moving on to Part II: Advanced.

Chapter 1

of

FRACTIONS, DECIMALS, & PERCENTS

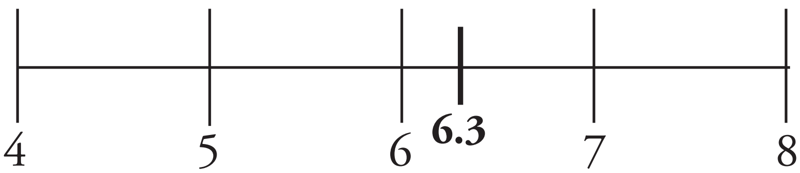
DIGITS & DECIMALS

In This Chapter . . .

- Place Value
- Using Place Value on the GMAT
- Rounding to the Nearest Place Value
- Adding Zeroes to Decimals
- Powers of 10: Shifting the Decimal
- The Last Digit Shortcut
- The Heavy Division Shortcut
- Decimal Operations

DECIMALS

GMAT math goes beyond an understanding of the properties of integers (which include the counting numbers, such as 1, 2, 3, their negative counterparts, such as –1, –2, –3, and 0). The GMAT also tests your ability to understand the numbers that fall in between the integers. Such numbers can be expressed as decimals. For example, the decimal 6.3 falls between the integers 6 and 7.



You can use a number line to decide between which whole numbers a decimal falls.

Some other examples of decimals include:

Decimals less than –1:	–3.65, –12.01, –145.9
Decimals between –1 and 0:	–0.65, –0.8912, –0.076
Decimals between 0 and 1:	0.65, 0.8912, 0.076
Decimals greater than 1:	3.65, 12.01, 145.9

Note that an integer can be expressed as a decimal by adding the decimal point and the digit 0. For example:

8 = 8.0   –123 = –123.0   400 = 400.0

DIGITS

Every number is composed of digits. There are only ten digits in our number system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The term digit refers to one building block of a number; it does not refer to a number itself. For example: 356 is a number composed of three digits: 3, 5, and 6.

Integers can be classified by the number of digits they contain. For example:

2, 7, and –8 are each single-digit numbers (they are each composed of one digit).
43, 63, and –14 are each double-digit numbers (composed of two digits).
500,000 and –468,024 are each six-digit numbers (composed of six digits).
789,526,622 is a nine-digit number (composed of nine digits).

Non-integers are not generally classified by the number of digits they contain, since you can always add any number of zeroes at the end, on the right side of the decimal point:

9.1 = 9.10 = 9.100

Place Value

Every digit in a number has a particular place value depending on its location within the number. For example, in the number 452, the digit 2 is in the ones (or "units") place, the digit 5 is in the tens place, and the digit 4 is in the hundreds place. The name of each location corresponds to the "value" of that place. Thus:

> 2 is worth two "units" (two "ones"), or 2 (= 2×1).
> 5 is worth five tens, or 50 (= 5×10).
> 4 is worth four hundreds, or 400 (= 4×100).

We can now write the number 452 as the **sum** of these products:

$$452 = 4 \times 100 \ + \ 5 \times 10 \ + \ 2 \times 1$$

You should memorize the names of all the place values.

6	9	2	5	6	7	8	9	1	0	2	3	.	8	3	4	7
HUNDRED BILLIONS	TEN BILLIONS	ONE BILLIONS	HUNDRED MILLIONS	TEN MILLIONS	ONE MILLIONS	HUNDRED THOUSANDS	TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	UNITS OR ONES		TENTHS	HUNDREDTHS	THOUSANDTHS	TEN THOUSANDTHS

The chart to the left analyzes the place value of all the digits in the number:

692,567,891,023.8347

Notice that the place values to the left of the decimal all end in "-s," while the place values to the right of the decimal all end in "-ths." This is because the suffix "-ths" gives these places (to the right of the decimal) a fractional value.

Let us analyze the end of the preceding number: **0.8347**

8 is in the tenths place, giving it a value of 8 tenths, or $\frac{8}{10}$.

3 is in the hundredths place, giving it a value of 3 hundredths, or $\frac{3}{100}$.

4 is in the thousandths place, giving it a value of 4 thousandths, or $\frac{4}{1000}$.

7 is in the ten thousandths place, giving it a value of 7 ten thousandths, or $\frac{7}{10{,}000}$.

To use a concrete example, 0.8 might mean eight tenths of one dollar, which would be 8 dimes or 80 cents. Additionally, 0.03 might mean three hundredths of one dollar, which would be 3 pennies or 3 cents.

Using Place Value on the GMAT

Some difficult GMAT problems require the use of place value with unknown digits.

> A and B are both two–digit numbers, with $A > B$. If A and B contain the same digits, but in reverse order, what integer must be a factor of $(A - B)$?
>
> (A) 4 (B) 5 (C) 6 (D) 8 (E) 9

To solve this problem, assign two variables to be the digits in A and B: x and y.
Let $A = \boxed{x}\boxed{y}$ (**not** the product of x and y: x is in the tens place, and y is in the units place). The boxes remind you that x and y stand for digits. A is therefore the sum of x tens and y ones. Using algebra, we write $A = 10x + y$.

Since B's digits are reversed, $B = \boxed{y}\boxed{x}$. Algebraically, B can be expressed as $10y + x$. The difference of A and B can be expressed as follows:

Place value can help you solve tough problems about digits.

$$A - B = 10x + y - (10y + x) = 9x - 9y = 9(x - y)$$

Clearly, 9 must be a factor of $A - B$. The correct answer is **(E)**.

You can also make up digits for x and y and plug them in to create A and B. This will not necessarily yield the unique right answer, but it should help you eliminate wrong choices.

In general, for unknown digits problems, be ready to create variables (such as x, y, and z) to represent the unknown digits. Recognize that each unknown is restricted to at most 10 possible values (0 through 9). Then apply any given constraints, which may involve number properties such as divisibility or odds & evens.

Rounding to the Nearest Place Value

The GMAT occasionally requires you to round a number to a specific place value.

> What is 3.681 rounded to the nearest tenth?

First, find the digit located in the specified place value. The digit 6 is in the tenths place.

Second, look at the right-digit-neighbor (the digit immediately to the right) of the digit in question. In this case, 8 is the right-digit-neighbor of 6. If the right-digit-neighbor is 5 or greater, round the digit in question UP. Otherwise, leave the digit alone. In this case, since 8 is greater than five, the digit in question (6) must be rounded up to 7. Thus, 3.681 rounded to the nearest tenth equals 3.7. Note that all the digits to the right of the right-digit-neighbor are irrelevant when rounding.

Rounding appears on the GMAT in the form of questions such as this:

> If x is the decimal 8.1d5, with d as an unknown digit, and x rounded to the nearest tenth is equal to 8.1, which digits could not be the value of d?

In order for x to be 8.1 when rounded to the nearest tenth, the right-digit-neighbor, d, must be less than 5. Therefore d cannot be 5, 6, 7, 8 or 9.

Adding Zeroes to Decimals

Adding zeroes to the end of a decimal or taking zeroes away from the end of a decimal does not change the value of the decimal. For example: $3.6 = 3.60 = 3.6000$

Be careful, however, not to add or remove any zeroes from within a number. Doing so will change the value of the number: $7.01 \neq 7.1$

Powers of 10: Shifting the Decimal

When you shift the decimal to the right, the number gets bigger. When you shift the decimal to the left, the number gets smaller.

Place values continually decrease from left to right by powers of 10. Understanding this can help you understand the following shortcuts for multiplication and division.
When you multiply any number by a positive power of ten, move the decimal forward (right) the specified number of places. This makes positive numbers larger:

In words	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
In numbers	1000	100	10	1	0.1	0.01	0.001
In powers of ten	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

$3.9742 \times 10^3 = 3{,}974.2$ (Move the decimal forward 3 spaces.)
$89.507 \times 10 = 895.07$ (Move the decimal forward 1 space.)

When you divide any number by a positive power of ten, move the decimal backward (left) the specified number of places. This makes positive numbers smaller:

$4{,}169.2 \div 10^2 = 41.692$ (Move the decimal backward 2 spaces.)
$89.507 \div 10 = 8.9507$ (Move the decimal backward 1 space.)

Note that if you need to add zeroes in order to shift a decimal, you should do so:

$2.57 \times 10^6 = 2{,}570{,}000$ (Add 4 zeroes at the end.)
$14.29 \div 10^5 = 0.0001429$ (Add 3 zeroes at the beginning.)

Finally, note that negative powers of ten reverse the regular process:

$6{,}782.01 \times 10^{-3} = 6.78201$ $53.0447 \div 10^{-2} = 5{,}304.47$

You can think about these processes as **trading decimal places for powers of ten**.

For instance, all of the following numbers equal 110,700.

110.7	×	10^3
11.07	×	10^4
1.107	×	10^5
0.1107	×	10^6
0.01107	×	10^7

The first number gets smaller by a factor of 10 as we move the decimal one place to the left, but the second number gets bigger by a factor of 10 to compensate.

The Last Digit Shortcut

Sometimes the GMAT asks you to find a units digit, or a remainder after division by 10.

What is the units digit of $(7)^2(9)^2(3)^3$?

In this problem, you can use the Last Digit Shortcut:

To find the units digit of a product or a sum of integers, only pay attention to the units digits of the numbers you are working with. Drop any other digits.

This shortcut works because only units digits contribute to the units digit of the product.

STEP 1: $7 \times 7 = 49$ Drop the tens digit and keep only the last digit: 9.
STEP 2: $9 \times 9 = 81$ Drop the tens digit and keep only the last digit: 1.
STEP 3: $3 \times 3 \times 3 = 27$ Drop the tens digit and keep only the last digit: 7.
STEP 4: $9 \times 1 \times 7 = 63$ Multiply the last digits of each of the products.

The units digit of the final product is 3.

Use the Heavy Division Shortcut when you need an approximate answer.

The Heavy Division Shortcut

Some division problems involving decimals can look rather complex. But sometimes, you only need to find an approximate solution. In these cases, you often can save yourself time by using the Heavy Division Shortcut: move the decimals in the same direction and round to whole numbers.

What is $1{,}530{,}794 \div (31.49 \times 10^4)$ to the nearest whole number?

Step 1: Set up the division problem in fraction form: $\frac{1{,}530{,}794}{31.49 \times 10^4}$

Step 2: Rewrite the problem, eliminating powers of 10: $\frac{1{,}530{,}794}{314{,}900}$

Step 3: Your goal is to get a single digit to the left of the decimal in the denominator. In this problem, you need to move the decimal point backward 5 spaces. You can do this to the denominator as long as you do the same thing to the numerator. (Technically, what you are doing is dividing top and bottom by the same power of 10: 100,000)

$$\frac{1{,}530{,}794}{314{,}900} = \frac{15.30794}{3.14900}$$

Now you have the single digit 3 to the left of the decimal in the denominator.

Step 4: Focus only on the whole number parts of the numerator and denominator and solve.

$$\frac{15.30794}{3.14900} \cong \frac{15}{3} = 5$$

An approximate answer to this complex division problem is 5. If this answer is not precise enough, keep one more decimal place and do long division (eg., $153 \div 31 \approx 4.9$).

Decimal Operations

<u>ADDITION AND SUBTRACTION</u>
To add or subtract decimals, make sure to line up the decimal points. Then add zeroes to make the right sides of the decimals the same length.

4.319 + 221.8		**10 – 0.063**	
Line up the decimal points and add zeroes.	4.319 + 221.800 226.119	Line up the decimal points and add zeroes.	10.000 – 0.063 9.937

Addition & Subtraction: Line up the decimal points!

The rules for decimal operations are different for each operation.

<u>MULTIPLICATION</u>
To multiply decimals, ignore the decimal point until the end. Just multiply the numbers as you would if they were whole numbers. Then count the *total* number of digits to the right of the decimal point in the factors. The product should have the same number of digits to the right of the decimal point.

0.02 × 1.4 Multiply normally:

$$\begin{array}{r} 14 \\ \times\, 2 \\ \hline 28 \end{array}$$

There are 3 digits to the right of the decimal point in the factors (0 and 2 in the first factor and 4 in the second factor). Therefore, move the decimal point 3 places to the left in the product: 28 → 0.028.

Multiplication: In the factors, count all the digits to the right of the decimal point—then put that many digits to the right of the decimal point in the product.

If the product ends with 0, count it in this process: 0.8 × 0.5 = 0.40, since 8 × 5 = 40.

If you are multiplying a very large number and a very small number, the following trick works to simplify the calculation: move the decimals **in the opposite direction** the same number of places.

0.0003 × 40,000 = ?

Move the decimal point RIGHT four places on the 0.0003 → 3
Move the decimal point LEFT four places on the 40,000 → 4

0.0003 × 40,000 = 3 × 4 = 12

The reason this technique works is that you are multiplying and then dividing by the same power of ten. In other words, you are **trading decimal places** in one number for decimal places in another number. This is just like trading decimal places for powers of ten, as we saw earlier.

DIVISION
If there is a decimal point in the dividend (the inner number) only, you can simply bring the decimal point straight up to the answer and divide normally.
Ex. **12.42 ÷ 3** = 4.14

```
   4.14
3)12.42
  12
   04
    3
    12
```

However, if there is a decimal point in the divisor (the outer number), you should shift the decimal point in both the divisor and the dividend to make the *divisor* a whole number. Then, bring the decimal point up and divide.

Remember, in order to divide decimals, you must make the OUTER number a whole number by shifting the decimal point.

Ex: **12.42÷ 0.3** → 124.2 ÷ 3 = 41.4

```
   41.4
3)124.2
  12
   04
    3
    12
```

Move the decimal one space to the right to make 0.3 a whole number. Then, move the decimal one space in 12.42 to make it 124.2.

Division: Divide by whole numbers!

You can always simplify division problems that involve decimals by shifting the decimal point **in the same direction** in both the divisor and the dividend, even when the division problem is expressed as a fraction:

$$\frac{0.0045}{0.09} = \frac{45}{900}$$

Move the decimal 4 spaces to the right to make both the numerator and the denominator whole numbers.

Note that this is essentially the same process as simplifying a fraction. You are simply multiplying the numerator and denominator of the fraction by a power of ten—in this case, 10^4, or 10,000.

Keep track of how you move the decimal point! To simplify multiplication, you can move decimals in opposite directions. But to simplify division, you move decimals in the same direction.

Equivalently, by adding zeroes, you can express the numerator and the denominator as the same units, then simplify:

$$\frac{0.0045}{0.09} = \frac{0.0045}{0.0900} = 45 \text{ ten thousandths} \div 900 \text{ ten–thousandths} = \frac{45}{900} = \frac{5}{100} = 0.05$$

POWERS AND ROOTS

To square or cube a decimal, you can always simply multiply it by itself once or twice. However, to raise a decimal to a larger power, you can rewrite the decimal as the product of an integer and a power of ten, and then apply the exponent.

$(0.5)^4 = ?$

Rewrite the decimal: $0.5 = 5 \times 10^{-1}$

Apply the exponent to each part: $(5 \times 10^{-1})^4 = 5^4 \times 10^{-4}$

Compute the first part and combine: $5^4 = 25^2 = 625$
$625 \times 10^{-4} = 0.0625$

Take a power or a root of a decimal by splitting the decimal into 2 parts: an integer and a power of ten.

Solve for roots of decimals the same way. Recall that a root is a number raised to a fractional power: a square root is a number raised to the 1/2 power, a cube root is a number raised to the 1/3 power, etc.

$\sqrt[3]{0.000027} = ?$

Rewrite the decimal. Make the first number something you can take the cube root of easily:

$0.000027 = 27 \times 10^{-6}$

Write the root as a fractional exponent: $(0.000027)^{1/3} = (27 \times 10^{-6})^{1/3}$

Apply the exponent to each part: $(27)^{1/3} \times (10^{-6})^{1/3} = (27)^{1/3} \times 10^{-2}$

Compute the first part and combine: $(27)^{1/3} = 3$ (since $3^3 = 27$)
$3 \times 10^{-2} = 0.03$

Powers and roots: Rewrite the decimal using powers of ten!

Once you understand the principles, you can take a shortcut by counting decimal places. For instance, the number of decimal places in the result of a cubed decimal is 3 times the number of decimal places in the original decimal:

$(0.04)^3 = 0.000064$ $\qquad$ $(0.04)^3$ = 0.000064
2 places $\qquad$ *$2 \times 3 = 6$ places*

Likewise, the number of decimal places in a cube root is 1/3 the number of decimal places in the original decimal:

$\sqrt[3]{0.000000008} = 0.002$ $\qquad$ $\sqrt[3]{0.000000008}$ = 0.002
9 places $\qquad$ *$9 \div 3 = 3$ places*

However, make sure that you can work with powers of ten using exponent rules.

Problem Set

Solve each problem, applying the concepts and rules you learned in this section.

1. What is the units digit of $(2)^5(3)^3(4)^2$?

2. What is the sum of all the possible 3-digit numbers that can be constructed using the digits 3, 4, and 5, if each digit can be used only once in each number?

3. In the decimal, 2.4*d*7, *d* represents a digit from 0 to 9. If the value of the decimal rounded to the nearest tenth is less than 2.5, what are the possible values of *d*?

4. If *k* is an integer, and if 0.02468×10^k is greater than 10,000, what is the least possible value of *k*?

5. Which integer values of *b* would give the number $2002 \div 10^{-b}$ a value between 1 and 100?

6. Estimate to the nearest 10,000: $\dfrac{4{,}509{,}982{,}344}{5.342 \times 10^4}$

7. Simplify: $(4.5 \times 2 + 6.6) \div 0.003$

8. Simplify: $(4 \times 10^{-2}) - (2.5 \times 10^{-3})$

9. What is $4{,}563{,}021 \div 10^5$, rounded to the nearest whole number?

10. Simplify: $(0.08)^2 \div 0.4$

11. Data Sufficiency: The number *A* is a two-digit positive integer; the number *B* is the two-digit positive integer formed by reversing the digits of *A*. If $Q = 10B - A$, what is the value of *Q*?

    (1) The tens digit of *A* is 7.
    (2) The tens digit of *B* is 6.

12. Simplify: $[8 - (1.08 + 6.9)]^2$

13. Which integer values of *j* would give the number $-37{,}129 \times 10^j$ a value between -100 and -1?

1. **4:** Use the Last Digit Shortcut, ignoring all digits but the last in any intermediate products:

STEP ONE: $2^5 = 32$ Drop the tens digit and keep only the last digit: 2.
STEP TWO: $3^3 = 27$ Drop the tens digit and keep only the last digit: 7.
STEP THREE: $4^2 = 16$ Drop the tens digit and keep only the last digit: 6.
STEP FOUR: $2 \times 7 \times 6 = 84$ Drop the tens digit and keep only the last digit: 4.

2. **2664:** There are 6 ways in which to arrange these digits: 345, 354, 435, 453, 534, and 543. Notice that each digit appears twice in the hundreds column, twice in the tens column, and twice in the ones column. Therefore, you can use your knowledge of place value to find the sum quickly:

$$100(24) + 10(24) + (24) = 2400 + 240 + 24 = 2664.$$

3. **{0, 1, 2, 3, 4}:** If d is 5 or greater, the decimal rounded to the nearest tenth will be 2.5.

4. **6:** Multiplying 0.02468 by a positive power of ten will shift the decimal point to the right. Simply shift the decimal point to the right until the result is greater than 10,000. Keep track of how many times you shift the decimal point. Shifting the decimal point 5 times results in 2,468. This is still less than 10,000. Shifting one more place yields 24,680, which is greater than 10,000.

5. **{−2, −3}:** In order to give 2002 a value between 1 and 100, we must shift the decimal point to change the number to 2.002 or 20.02. This requires a shift of either two or three places to the left. Remember that, while multiplication shifts the decimal point to the right, division shifts it to the left. To shift the decimal point 2 places to the left, we would divide by 10^2. To shift it 3 places to the left, we would divide by 10^3. Therefore, the exponent $-b = \{2, 3\}$, and $b = \{-2, -3\}$.

6. **90,000:** Use the Heavy Division Shortcut to estimate:

$$\frac{4{,}509{,}982{,}344}{53{,}420} \approx \frac{4{,}500{,}000{,}000}{50{,}000} = \frac{450{,}000}{5} = 90{,}000$$

7. **5,200:** Use the order of operations, PEMDAS (Parentheses, Exponents, Multiplication & Division, Addition and Subtraction) to simplify.

$$\frac{9 + 6.6}{0.003} = \frac{15.6}{0.003} = \frac{15{,}600}{3} = 5{,}200$$

8. **0.0375:** First, rewrite the numbers in standard notation by shifting the decimal point. Then, add zeroes, line up the decimal points, and subtract.

$$\begin{array}{r} 0.0400 \\ -\ \underline{0.0025} \\ 0.0375 \end{array}$$

9. **46:** To divide by a positive power of 10, shift the decimal point to the left. This yields 45.63021. To round to the nearest whole number, look at the tenths place. The digit in the tenths place, 6, is more than five. Therefore, the number is closest to 46.

10. **0.016:** Use the order of operations, PEMDAS (Parentheses, Exponents, Multiplication & Division, Addition and Subtraction) to simplify. Shift the decimals in the numerator and denominator so that you are dividing by an integer.

$$\frac{(0.08)^2}{0.4} = \frac{0.0064}{0.4} = \frac{0.064}{4} = 0.016$$