EQUATIONS, INEQUALITIES, & VICs

Math Strategy Guide

This essential guide covers algebra in all its various forms (and disguises) on the GMAT. Master fundamental techniques and nuanced strategies to help you solve for unknown variables of every type.

Equations, Inequalities, and VICs GMAT Strategy Guide, Fourth Edition

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Dear Student,

Thank you for picking up one of the Manhattan GMAT Strategy Guides—we hope that it refreshes your memory of the junior-high math that you haven't used in years. Maybe it will even teach you a new thing or two.

As with most accomplishments, there were many people involved in the various iterations of the book that you're holding. First and foremost is Zeke Vanderhoek, the founder of Manhattan GMAT. Zeke was a lone tutor in New York when he started the Company in 2000. Now, nine years later, MGMAT has Instructors and offices nationwide, and the Company contributes to the studies and successes of thousands of students each year.

Our 4th Edition Strategy Guides are based on the continuing experiences of our Instructors and our students. We owe much of these latest editions to the insight provided by our students. On the Company side, we are indebted to many of our Instructors, including but not limited to Josh Braslow, Dan Gonzalez, Mike Kim, Stacey Koprince, Ben Ku, Jadran Lee, David Mahler, Ron Purewal, Tate Shafer, Emily Sledge, and of course Chris Ryan, the Company's Lead Instructor and Director of Curriculum Development.

At Manhattan GMAT, we continually aspire to provide the best Instructors and resources possible. We hope that you'll find our dedication manifest in this book. If you have any comments or questions, please e-mail me at andrew.yang@manhattangmat.com. I'll be sure that your comments reach Chris and the rest of the team—and I'll read them too.

Best of luck in preparing for the GMAT!

Sincerely,

Andrew Yang
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PART I:
GENERAL

TABLE OF CONTENTS

PART II:
ADVANCED

TABLE OF CONTENTS

Chapter 1

of

EQUATIONS, INEQUALITIES, & VICs

BASIC EQUATIONS

In This Chapter . . .

- Solving One-Variable Equations
- Simultaneous Equations: Solving by Substitution
- Simultaneous Equations: Solving by Combination
- Simultaneous Equations: Three Equations
- Mismatch Problems
- Combo Problems: Manipulations
- Testing Combos in Data Sufficiency
- Absolute Value Equations

BASIC EQUATIONS

Algebra is one of the major math topics tested on the GMAT. Your ability to solve equations is an essential component of your success on the exam.

Basic GMAT equations are those that DO NOT involve exponents. The GMAT expects you to solve several different types of BASIC equations :

1) An equation with 1 variable
2) Simultaneous equations with 2 or 3 variables
3) Mismatch equations
4) Combos
5) Equations with absolute value

To solve basic equations, remember that whatever you do to one side, you must also do to the other side.

Several of the preceding basic equation types probably look familiar to you. Others—particularly Mismatch Equations and Combos—are unique GMAT favorites that run counter to some of the rules you may have learned in high-school algebra. Becoming attuned to the particular subtleties of GMAT equations can be the difference between an average score and an excellent one.

Solving One-Variable Equations

Equations with one variable should be familiar to you from previous encounters with algebra. In order to solve one-variable equations, simply isolate the variable on one side of the equation. In doing so, make sure you perform identical operations to both sides of the equation. Here are three examples:

$3x + 5 = 26$ Subtract 5 from both sides.
$3x = 21$ Divide both sides by 3.
$x = 7$

$w = 17w - 1$ Subtract w from both sides.
$0 = 16w - 1$ Add 1 to both sides.
$1 = 16w$ Divide both sides by 16.
$\frac{1}{16} = w$

$\frac{p}{9} + 3 = 5$ Subtract 3 from both sides.

$\frac{p}{9} = 2$ Multiply both sides by 9.

$p = 18$

Simultaneous Equations: Solving by Substitution

Sometimes the GMAT asks you to solve a system of equations with more than one variable. You might be given two equations with two variables, or perhaps three equations with three variables. In either case, there are two primary ways of solving simultaneous equations: by substitution or by combination.

Solve the following for x and y.

$$x + y = 9$$
$$2x = 5y + 4$$

Use substitution whenever one variable can be easily expressed in terms of the other.

1. Solve the first equation for x.

$$x + y = 9$$
$$x = 9 - y$$

2. Substitute this solution into the second equation wherever x appears.

$$2x = 5y + 4$$
$$2(9 - y) = 5y + 4$$

3. Solve the second equation for y.

$$2(9 - y) = 5y + 4$$
$$18 - 2y = 5y + 4$$
$$14 = 7y$$
$$2 = y$$

4. Substitute your solution for y into the first equation in order to solve for x.

$$x + y = 9$$
$$x + 2 = 9$$
$$x = 7$$

Simultaneous Equations: Solving by Combination

Alternatively, you can solve simultaneous equations by combination. In this method, add or subtract the two equations to eliminate one of the variables.

Solve the following for x and y.

$$x + y = 9$$
$$2x = 5y + 4$$

1. Line up the terms of the equations.

$$x + y = 9$$
$$2x - 5y = 4$$

2. If you plan to add the equations, multiply one or both of the equations so that the coefficient of a variable in one equation is the OPPOSITE of that variable's coefficient in the other equation. If you plan to subtract them, multiply one or both of the equations so that the coefficient of a variable in one equation is the SAME as that variable's coefficient in the other equation.

$$-2(x + y = 9) \rightarrow -2x - 2y = -18$$
$$2x - 5y = 4 \rightarrow 2x - 5y = 4$$

Note that the x coefficients are now opposites.

Use combination whenever it is easy to manipulate the equations so that the coefficients for one variable are the SAME or OPPOSITE.

3. Add the equations to eliminate one of the variables.

$$\begin{array}{rr} & -2x - 2y = -18 \\ + & 2x - 5y = 4 \\ \hline & -7y = -14 \end{array}$$

4. Solve the resulting equation for the unknown variable.

$$-7y = -14$$
$$y = 2$$

5. Substitute into one of the original equations to solve for the second variable.

$$x + y = 9$$
$$x + 2 = 9$$
$$x = 7$$

Simultaneous Equations: Three Equations

The procedure for solving a system of three equations with three variables is exactly the same as for a system with two equations and two variables. You can use substitution or combination. This example uses both:

Solve the following for w, x, and y.

$$x + w = y$$
$$2y + w = 3x - 2$$
$$13 - 2w = x + y$$

Solve three simultaneous equations step-by-step. Keep careful track of your work to avoid careless errors, and look for ways to reduce the number of steps needed to solve.

1. The first equation is already solved for y.

$$y = x + w$$

2. Substitute for y in the second and third equations.

Substitute for y in the second equation:

$$2(x + w) + w = 3x - 2$$
$$2x + 2w + w = 3x - 2$$
$$-x + 3w = -2$$

Substitute for y in the third equation:

$$13 - 2w = x + (x + w)$$
$$13 - 2w = 2x + w$$
$$3w + 2x = 13$$

3. Multiply the first of the resulting two-variable equations by (–1) and combine them with addition.

$$\begin{array}{rr} & x - 3w = \ \ 2 \\ + & 2x + 3w = 13 \\ \hline & 3x = 15 \end{array}$$

Therefore, $x = 5$

4. Use your solution for x to determine solutions for the other two variables.

$$3w + 2x = 13$$
$$3w + 10 = 13$$
$$3w = 3$$
$$w = 1$$

$$y = x + w$$
$$y = 5 + 1$$
$$y = 6$$

The preceding example requires a lot of steps to solve. Therefore it is unlikely that the GMAT will ask you to solve such a complex system—it would be difficult to complete in two minutes. Here is the key to handling systems of three or more equations on the GMAT: look for ways to simplify the work. Look especially for shortcuts or symmetries in the form of the equations to reduce the number of steps needed to solve the system.

Take this system as an example:

What is the sum of x, y and z?

$$x + y = 8$$
$$x + z = 11$$
$$y + z = 7$$

In this case, DO NOT try to solve for x, y, and z individually. Instead, notice the symmetry of the equations—each one adds exactly two of the variables—and add them all together:

$$\begin{array}{rrrl} x + & y & & = 8 \\ x & & + z & = 11 \\ + & + y & + z & = 7 \\ \hline 2x + & 2y & + 2z & = 26 \end{array}$$

Therefore, $x + y + z$ is half of 26, or 13.

Do not assume that the number of equations must be equal to the number of variables.

Mismatch Problems

Consider the following rule, which you might have learned in a basic algebra course: if you are trying to solve for 2 different variables, you need 2 equations. If you are trying to solve for 3 different variables, you need 3 equations, etc. The GMAT loves to trick you by taking advantage of your faith in this easily misapplied rule.

MISMATCH problems, which are particularly common on the Data Sufficiency portion of the test, are those in which the number of unknown variables does NOT correspond to the number of given equations. Do not try to apply that old rule you learned in high-school algebra. All MISMATCH problems must be solved on a case-by-case basis. Try the following Data Sufficiency problem:

What is x?

(1) $\frac{3x}{3y + 5z} = 8$ (2) $6y + 10z = 18$

It is tempting to say that these two equations are not sufficient to solve for x, since there are 3 variables and only 2 equations. However, the question does NOT ask you to solve for all three variables. It only asks you to solve for x, which IS possible:

First, get the x term on one side of the equation:

$$\frac{3x}{3y + 5z} = 8$$
$$3x = 8(3y + 5z)$$

Then, notice that the second equation gives us a value for $3y + 5z$, which we can substitute into the first equation in order to solve for x:

$$6y + 10z = 18$$
$$2(3y + 5z) = 18$$
$$3y + 5z = 9$$

$$3x = 8(3y + 5z)$$
$$3x = 8(9)$$
$$x = 8(3) = 24$$

The answer is **(C)**: BOTH statements TOGETHER are sufficient.

Now consider an example in which 2 equations with 2 unknowns are actually NOT sufficient to solve a problem:

What is x?

(1) $y = x^3 - 1$ (2) $y = x - 1$

It is tempting to say that these 2 equations are surely sufficient to solve for x, since there are 2 different equations and only 2 variables. However, notice that if we take the expression for y in the first equation and substitute into the second, we actually get multiple answers:

Follow through with the algebra on potential mismatch problems to determine whether a single solution is possible.

$$x^3 - 1 = x - 1$$
$$x^3 = x$$
$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x+1)(x-1) = 0$$
$$x = \{-1, 0, 1\}$$

Because of the exponent on x, we have THREE values for x (−1, 0, or 1). Therefore we do NOT have sufficient information to solve for x, and the answer to the problem is **(E)**: the statements together are NOT sufficient. This example is typical.

Now consider another example in which 2 equations with 2 unknowns are actually NOT sufficient to solve a problem. This time we will avoid exponents altogether:

What is x?

(1) $x - y = 1$ (2) $xy = 12$

Again, it is tempting to say that these 2 equations are sufficient to solve for x, since there are 2 equations and only 2 variables. However, when you actually combine the two equations, you wind up with a quadratic equation that has multiple solutions:

$$x - y = 1$$
$$x - 1 = y$$
$$x(x-1) = 12$$
$$x^2 - x = 12$$
$$x^2 - x - 12 = 0$$
$$(x-4)(x+3) = 0$$
$$x = 4 \text{ or } x = -3$$

Although we have narrowed down the possibilities for x to just two choices, this is not enough. We do NOT have sufficient information to solve for x, and again, the answer to the problem is **(E)**: the statements together are NOT sufficient.

A MASTER RULE for determining whether 2 equations involving 2 variables (say, x and y) will be sufficient to solve for the variables is this:

(1) If both of the equations are linear—that is, if there are no squared terms (such as x^2 or y^2) and no xy terms—the equations will be sufficient UNLESS the two equations are mathematically identical (e.g., $x + y = 10$ is identical to $2x + 2y = 20$).

(2) If there are ANY non-linear terms in either of the equations (such as x^2, y^2, xy, or $\frac{x}{y}$), there will USUALLY be two (or more) different solutions for each of the variables and the equations will not be sufficient.

With 2 equations and 2 unknowns, linear equations usually lead to one solution and nonlinear equations usually lead to 2 (or more) solutions.

Examples:

What is x?

(1) $2x + 3y = 8$
(2) $2x - y = 0$

Because both of the equations are linear, and because they are not mathematically identical, there is only one solution ($x = 1$ and $y = 2$) so the statements are SUFFICIENT TOGETHER (answer **C**).

What is x?

(1) $x^2 + y = 17$
(2) $y = 2x + 2$

Because there is an x^2 term in equation 1, as usual there are two solutions for x and y ($x = 3$ and $y = 8$, or $x = -5$ and $y = -8$), so the statements are NOT SUFFICIENT, even together (answer **E**).

Combo Problems: Manipulations

The GMAT often asks you to solve for a combination of variables, called COMBO problems. For example, a question might ask, what is the value of $x + y$?

In these cases, since you are not asked to solve for one specific variable, you should generally NOT try to solve for the individual variables right away. Instead, you should try to manipulate the given equation(s) so that the COMBO is isolated on one side of the equation. Only try to solve for the individual variables after you have exhausted all other avenues.

There are four easy manipulations that are the key to solving most COMBO problems. You can use the acronym **MADS** to remember them.

M: Multiply or divide the whole equation by a certain number.
A: Add or subtract a number on both sides of the equation.
D: Distribute or factor an expression on ONE side of the equation.
S: Square or unsquare both sides of the equation.

Here are three examples, each of which uses one or more of these manipulations:

If $x = \frac{7-y}{2}$, what is $2x + y$?

$$x = \frac{7-y}{2}$$
$$2x = 7 - y$$
$$2x + y = 7$$

Here, getting rid of the denominator by multiplying both sides of the equation by 2 is the key to isolating the combo on one side of the equation.

If $\sqrt{2t + r} = 5$, what is $3r + 6t$?

$$\left(\sqrt{2t + r}\right)^2 = 5^2$$
$$2t + r = 25$$
$$6t + 3r = 75$$

Here, getting rid of the square root by squaring both sides of the equation is the first step. Then, multiplying the whole equation by 3 forms the combo in question.

To solve for a variable combo, isolate the combo on one side of the equation.

If $a(4 - c) = 2ac + 4a + 9$, what is ac?

$$4a - ac = 2ac + 4a + 9$$
$$-ac = 2ac + 9$$
$$-3ac = 9$$
$$ac = -3$$

Here, distributing the term on the left-hand side of the equation is the first key to isolating the combo on one side of the equation; then we have to subtract $2ac$ from both sides of the equation.

Testing Combos in Data Sufficiency

Combo problems occur most frequently in Data Sufficiency. Whenever you detect that a Data Sufficiency question may involve a combo, you should try to manipulate the given equation(s) in either the question or the statement, so that the combo is isolated on one side of the equation. Then, if the other side of an equation from a statement contains a VALUE, that equation is SUFFICIENT. If the other side of the equation contains a VARIABLE EXPRESSION, that equation is NOT SUFFICIENT.

What is $\dfrac{\frac{2}{y}}{\frac{4}{x}}$?

(1) $\dfrac{x+y}{y}=3$

(2) $x+y=12$

Avoid attempting to solve for the individual variables in a combo problem, unless there is no obvious alternative.

First, rephrase the question by manipulating the given expression:

$$\frac{\frac{2}{y}}{\frac{4}{x}}=? \qquad \frac{2}{y}\times\frac{x}{4}=\frac{2x}{4y}=\frac{x}{2y}=\frac{1}{2}\times\frac{x}{y}=?$$

Now, we can ignore the 1/2 and isolate the combo we are looking for:

$$\frac{x}{y}=?$$

Manipulate statement (1) to solve for $\frac{x}{y}$ on one side of the equation. Since the other side of the equation contains a VALUE, statement (1) is SUFFICIENT:

$$\frac{x+y}{y}=3 \qquad x=2y$$

$$x+y=3y \qquad \frac{x}{y}=2$$

Manipulate statement (2) to solve for $\frac{x}{y}$ on one side of the equation. Since the other side of the equation contains a VARIABLE EXPRESSION, Statement (2) is INSUFFICIENT:

$$x+y=12 \qquad \frac{x}{y}=\frac{12-y}{y}$$

$$x=12-y \qquad \frac{x}{y}=\frac{12}{y}-1$$

The key to solving this problem easily is to AVOID trying to solve for the individual variables.

Absolute Value Equations

Absolute value refers to the POSITIVE value of the expression within the absolute value brackets. Equations that involve absolute value generally have TWO SOLUTIONS, because the actual value of the expression inside the brackets could be POSITIVE OR NEGATIVE. It is important to consider this rule when thinking about GMAT questions that involve absolute value. The following three-step method should be used when solving for a variable expression inside absolute value brackets.

Do not forget to check each of your solutions to absolute value equations by putting each solution back into the original equation.

Solve for w, given that $12 + |w - 4| = 30$.

1. Isolate the expression within the absolute value brackets.

$$12 + |w - 4| = 30$$

$$|w - 4| = 18$$

2. The rule, once we have an equation of the form $|x| = a$ with $a > 0$, is that $x = \pm a$.

Remove the absolute value brackets and solve the equation for 2 cases:

CASE 1: $x = a$ (x is positive)	CASE 2: $x = -a$ (x is negative)
$w - 4 = 18$	$w - 4 = -18$
$w = 22$	$w = -14$

3. Check to see whether each solution is valid by putting each one back into the original equation and verifying that the two sides of the equation are in fact equal.

In case 1, the solution, $w = 22$, is valid because $12 + |22 - 4| = 12 + 18 = 30$.
In case 2, the solution, $w = -14$, is valid because $12 + |-14 - 4| = 12 + 18 = 30$.

Solve for n, given that $|n + 9| - 3n = 3$.

Again, isolate the expression within the absolute value brackets and consider both cases.

1. $|n + 9| = 3 + 3n$

2.

CASE 1: $n + 9$ is positive:	CASE 2: $n + 9$ is negative:
$n + 9 = 3 + 3n$	$n + 9 = -(3 + 3n)$
$n = 3$	$n = -3$

3. The first solution, $n = 3$, is valid because $|(3) + 9| - 3(3) = 12 - 9 = 3$.

However the second solution, $n = -3$, is NOT valid because $|(-3) + 9| - 3(-3) = 6 + 9 = 15$. This solution fails because when $n = -3$, the absolute value expression ($n + 9 = 6$) is not negative, even though we assumed it was negative when we calculated that solution.

Problem Set

For problems #1–5, solve for all unknowns.

1. $\frac{3x-6}{5} = x - 6$

2. $\frac{x+2}{4+x} = \frac{5}{9}$

3. $22 - |y + 14| = 20$

4. $y = 2x + 9$ and $7x + 3y = -51$

5. $a + b = 10$, $b + c = 12$, and $a + c = 16$

For problems #6–8, determine whether it is *possible* to solve for x using the given equations. (Do not solve.)

6. $\frac{\sqrt{x}}{6a} = T$ and $\frac{Ta}{4} = 14$

7. $3x + 2a = 8$ and $6a = 24 - 9x$

8. $3a + 2b + x = 8$ and $12a + 8b + 2x = 4$

For problems #9–12, solve for the specified expression.

9. Given that $\frac{x+y}{3} = 17$, what is $x + y$?

10. Given that $\frac{a+b}{b} = 21$, what is $\frac{a}{b}$?

11. Given that $10x + 10y = x + y + 81$, what is $x + y$?

12. Given that $\frac{b+a}{2a} = 2$ and $a + b = 8$, what is a?

For #13–15, write the expression in factored form (if distributed) and in distributed form (if factored):

13. $y^6 - y^4$

14. $5^6 - 5^5 + 5^4$

15. $(q + r)(s + t)$

1. ***x* = 12:**

$$\frac{3x-6}{5} = x - 6$$
$$3x - 6 = 5(x - 6)$$
$$3x - 6 = 5x - 30$$
$$24 = 2x$$
$$12 = x$$

Solve by multiplying both sides by 5 to eliminate the denominator. Then, distribute and isolate the variable on the left side.

2. ***x* = $\frac{1}{2}$:**

$$\frac{x+2}{4+x} = \frac{5}{9}$$
$$9(x + 2) = 5(4 + x)$$
$$9x + 18 = 20 + 5x$$
$$4x = 2$$
$$x = \frac{1}{2}$$

Cross-multiply to eliminate the denominators. Then, distribute and solve.

3. ***y* = {−16, −12}:**

$$22 - |y + 14| = 20$$
$$|y + 14| = 2$$

First, isolate the expression within the absolute value brackets. Then, solve for two cases, one in which the expression is positive and one in which it is negative. Finally, test the validity of your solutions.

Case 1: $y + 14 = 2$, $y = -12$

Case 2: $y + 14 = -2$, $y = -16$

Case 1 is valid because $22 - |-12 + 14| = 22 - 2 = 20$.
Case 2 is valid because $22 - |-16 + 14| = 22 - 2 = 20$.

4. ***x* = −6; *y* = −3:**

$$y = 2x + 9 \qquad 7x + 3y = -51$$
$$7x + 3(2x + 9) = -51$$
$$7x + 6x + 27 = -51$$
$$13x + 27 = -51$$
$$13x = -78$$
$$x = -6$$
$$y = 2x + 9 = 2(-6) + 9 = -3$$

Solve this system by substitution. Substitute the value given for y in the first equation into the second equation. Then, distribute, combine like terms, and solve. Once you get a value for x, substitute it back into the first equation to obtain the value of y.

5. ***a* = 7; *b* = 3; *c* = 9:** This problem could be solved by an elaborate series of substitutions. However, because the coefficients on each variable in each equation are equal to 1, combination proves easier. Here is one way, though certainly not the only way, to solve the problem: