NP

If both x and y are odd, is xy odd?

NP

Is the statement sufficient?

Is $x < 0$?

1) $xy^2 < 0$

NP

Odds & Evens

Answer: Yes, *xy* is odd

Odd numbers can be represented as $2m + 1$ or $2n + 1$, where m and n are integers. (Think about why this is.) Multiplying two numbers of this form together would yield $4nm + 2m + 2n + 1$, which is always odd; the 1st, 2nd, and 3rd terms are multiplied by 2 (or 4), so they are even, as is their sum. An even number plus one is odd. Thus xy is odd.

More simply, we could just recall: an odd number times an odd number is always odd.

When in doubt, try it out! Pick numbers to test properties.

NP

Positives & Negatives

Answer: Sufficient

Any number, except for 0, raised to an even power will be positive. If y were 0, the inequality would not be true, so we know that y^2, regardless of the sign of y, will be positive. For xy^2 to be less than zero, that means that x must be negative. The statement is sufficient.

NP

Simplify $\sqrt{6,300}$.

NP

If both x and y are odd, is $x^2 + y$ odd?

NP

Simplifying a Root

Answer: $30\sqrt{7}$

Whenever simplifying an expression under the square root sign, factor the expression. In this case, $6,300 = 2^2 \times 3^2 \times 5^2 \times 7$. For every pair under the square root sign, move one outside the radical, and throw the other away: $\sqrt{2^2 3^2 5^2 7}$ becomes $(2)(3)(5)\sqrt{7}$, or simply $30\sqrt{7}$.

NP

Odds & Evens

Answer: No, $x^2 + y$ is even

Odd numbers can be represented as $2m + 1$ or $2n + 1$, where m and n are integers. (Think about why this is.) $(2m + 1)^2 = 4m^2 + 4m + 1$, and adding $2n + 1$ would yield $4m^2 + 4m + 2n + 2$. This is always even, since a 2 can be factored from all four terms.

More simply, we could just recall: an odd number times an odd number is always odd, and an odd plus an odd is always even.

When in doubt, try it out! Pick numbers to test properties.

NP

If x is odd and y is even, is xy odd or even?

NP

Is the statement sufficient?

Is $x < 0$?

1) $\sqrt[13]{x} < 0$

NP

Odds & Evens

Answer: *xy* is even

An odd number can be represented as $2m + 1$, and an even number can be represented as $2n$, where m and n are integers. (Think about why this is.) Multiplying $2m + 1$ and $2n$ would yield $4mn + 2n$, which is always even, since 2 is a factor of both terms. (Factor out the 2 to get $2(2mn + n)$, which shows that this number will be even.)

More simply, we could just recall: an odd number times an even number is always even.

When in doubt, try it out! Pick numbers to test properties.

NP

Odd Roots

Answer: Sufficient

Don't let the 13 confuse you; the only thing that matters is that 13 is an odd number. Odd roots, as well as odd exponents, preserve the sign of the number inside. If $\sqrt[13]{x} < 0$, then x is also less than 0. The statement is sufficient.

NP

Simplify $\frac{85}{\sqrt{5}}$.

NP

If both x and y are odd, is $x - y$ odd?

NP

Simplifying a Root

Answer: $17\sqrt{5}$

When a square root lurks in the denominator, we can rationalize the denominator by multiplying by the appropriate form of 1 – in this case, $\frac{\sqrt{5}}{\sqrt{5}}$. $\left(\frac{85}{\sqrt{5}}\right)\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{85\sqrt{5}}{5}$, and 85 divided by 5 is 17, so the simplest form is $17\sqrt{5}$.

NP

Odds & Evens

Answer: No, $x - y$ is even

Odd numbers can be represented as $2m + 1$ or $2n + 1$, where m and n are integers. (Think about why this is.) Subtracting two numbers of this form would yield $(2n + 1) - (2m + 1)$, or just $2n - 2m$, which is always even, since a 2 can be factored out of the remaining terms (i.e., $2(n - m)$).

More simply, we could just recall: an odd number minus an odd number is always even.

When in doubt, try it out! Pick numbers to test properties.

NP

Calculate $(-1)^{789}$.

NP

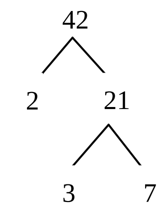
Exponents

Answer: –1

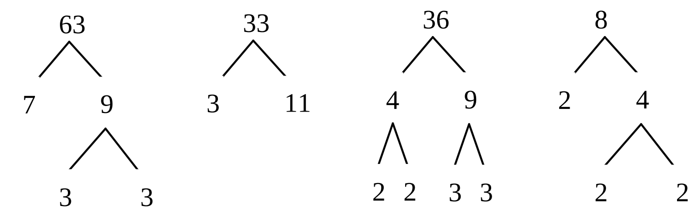
Since $(-1) \times (-1) = 1$, –1 raised to any even power is 1. If you multiply by –1 one more time, you end up with –1, so –1 raised to any odd power will equal –1. 789 is an odd number, so $(-1)^{789} = -1$.

NP

x is divisible by 42. Which of the following numbers is definitely a factor of x^2? (Choose all that apply.)



a) 63 b) 33 c) 36 d) 8



NP

Divisibility

Answer: a) 63 and c) 36

If x definitely has 2, 3 and 7 as factors, then when we square x, we know that x^2 will have two 2s, two 3s and two 7s as factors. 63 is $7 \times 3 \times 3$, and 36 is $2 \times 2 \times 3 \times 3$. Using the factor foundation rule, we can guarantee that all numbers that solely use those factors are factors of x^2. Both 63 and 36 use only prime factors found in x^2.

NP

$\left(\sqrt[5]{n}\right)^5$ is always equal to which of the following?

a) n
b) n^{25}
c) $n^{1/5}$
d) 1

NP

If both x and y are even, is $x - y$ even?

NP

Exponents & Roots

Answer: a) *n*

Try it: $\left(\sqrt[5]{n}\right)^5$ is the same as $\left(n^5\right)^{1/5}$, which is equal to n, since $(n^a)^b = n^{ab}$, and 5 times 1/5 equals 1.

Alternatively, $\left(\sqrt[5]{n}\right)^5 = \left(\sqrt[5]{n}\right)\left(\sqrt[5]{n}\right)\left(\sqrt[5]{n}\right)\left(\sqrt[5]{n}\right)\left(\sqrt[5]{n}\right) = n$.

You can try this out if you need convincing. Pick a few numbers and see what happens!

NP

Odds & Evens

Answer: Yes, $x - y$ is even

Even numbers can be represented as $2m$ or $2n$, where m and n are integers. (Think about why this is.) Subtracting two numbers of this form would yield $2n - 2m$, or $2(n - m)$, which has 2 as a factor, so it is even.

More simply, we could just recall: an even number minus an even number is always even.

When in doubt, try it out! Pick numbers to test properties.

NP

Is the statement sufficient?

$xy < 0$. Is $y < 0$?

1) $y^2\sqrt{x} > 0$

NP

If the units digit of an integer is 7, then which one-digit integers is it definitely NOT divisible by?

NP

Even Roots

Answer: Sufficient

If we know that $xy < 0$, then we know that x and y have different signs – one must be positive and the other negative. From the statement, we know that x must be positive, because we are not allowed to take an even root of a negative number. If x is positive, then y must be negative. The answer to the question is yes, and the statement is sufficient.

NP

Divisibility Rules

Answer: 2, 4, 5, 6, and 8

Integers that are divisible by 2, 4, 6, or 8 end in 2, 4, 6, 8, or 0; those divisible by 5 end in 5 or 0.

As an exercise, try to provide examples of integers with a ones digit of 7 that are divisible by 1, 3, 7, and 9.

NP

Calculate $16^{5/4}$.

NP

Is the statement sufficient?

If *x* is divisible by *y*, is *x/y* odd?

1) *x* and *y* are both odd.

NP

Fractional Exponents

Answer: 32

Using the rules of exponents, $16^{5/4} = \left(16^5\right)^{1/4} = \left(16^{1/4}\right)^5$.

Since it is easier to calculate $16^{1/4}$ than it is to calculate 16^5, the latter representation will be easier to simplify. $16^{1/4} = 2$, and $2^5 = 32$.

NP

Odds and Evens

Answer: Sufficient

This question is tricky, because an odd divided by an odd can yield an odd integer *or a non-integer*. However, the question stem states that *x* is divisible by *y*. Therefore, *x*/*y* is an integer, and the result must be odd.

NP

If an integer that is divisible by 6 is squared, then which (nonzero) one-digit integers is this squared result definitely divisible by?

NP

$$\frac{(6^4)(50^3)}{(2^4)(3^4)(10^3)} =$$

NP

Divisibility

Answer: 1, 2, 3, 4, 6, and 9

Call the original integer n. Since n is divisible by 6, we can say $n = 6m$, where m is any integer. Squaring n yields $n^2 = (6m)^2 = 36m^2$. Since 36 is divisible by 1, 2, 3, 4, 6, and 9, they are all factors of n^2 as well.

Any combination of 5, 7, and/or 8 may also divide n^2, but we can't say for sure whether they do without knowing what m is.

NP

Simplifying Exponential Expressions

Answer: 5^3

Instead of multiplying out everything, look for ways to reduce. On the top of the fraction, 6^4 can be separated into $(2^4)(3^4)$. This can be cancelled with the 2^4 and 3^4 on the bottom of the fraction, so we are left with $\frac{50^3}{10^3}$, which can be reduced to 5^3. (Note that $5^3 = 125$.)

NP

Simplify the following expression:

$(4(6(8(9^0))^1)^{-1})^2$

NP

If the ones digit of an integer is 0, then which (nonzero) one-digit integers is the integer definitely NOT divisible by?

NP

Order of Operations

Answer: 1/144

PEMDAS dictates the order of operations to perform. We must always calculate the innermost parentheses first, then work our way outwards. Calculate $9^0 = 1$ first; then $8^1 = 8$. Next we have $(6(8))^{-1} = 1/48$.; then $(4/48)^2 = 1/144$.

It's easy to remember PEMDAS with this saying: Please Excuse My Dear Aunt Sally!

NP

Divisibility Rules

Answer: None

It could be divisible by any of the one-digit integers! (Except for 0; dividing by 0 is always off limits.)

To verify, take any nonzero one-digit integer, multiply it by ten, and the product will end in zero and be divisible by the original one-digit integer.

NP

If x is even and y is odd, is $x^2 + y^2$ even or odd?

NP

Given that $y^7 < y^6$, describe all of the possible values for y.

NP

Odds & Evens

Answer: $x^2 + y^2$ is odd

An even number can be represented as $2m$, and an odd number can be represented as $2n + 1$, where m and n are integers. Squaring the even number yields $4m^2$; the odd, $4n^2 + 4n + 1$. Adding these together yields $4m^2 + 4n^2 + 4n + 1$. The 1st 3 terms all have 4 as a factor, so their sum is even, and an even number plus 1 is odd.

More simply, we could just recall: an even number squared is always even, an odd number squared is always odd, and an even plus an odd is always odd.

When in doubt, try it out! Pick numbers to test properties.

NP

Testing Positive and Negative Cases

Answer: $y < 1$, but not equal to 0 (alternatively, $0 < y < 1$ or $y < 0$)

Think about various categories of numbers: if y were negative, then y^7 would also be negative, while y^6 would be positive; then $y^7 < y^6$. If $y = 0$ or 1, then $y^7 = y^6$, which is not acceptable. When y is between 0 and 1, $y^7 < y^6$, since y^7 would equal y^6 times some fraction between 0 and 1. Finally, when $y > 1$, $y^7 > y^6$.

NP

Is the statement sufficient?

The positive integer x is a prime number. What is x?

1) $x + 11$ is a prime number.

NP

Is the statement sufficient?

Is $a < 0$?

1) $a^b < 0$

NP

Prime Numbers

Answer: Sufficient

If you tested numbers to answer this question, you probably figured out pretty quickly that 2 is a possible value of x. If you continue to test numbers to make sure there are no other possible values for x, you may notice a pattern emerging. $11 + 3 = 14$, $11 + 5 = 16$, $11 + 7 = 18$, etc. 11 plus any prime besides 2 will yield an even number. 2 is the only even prime, because every other even number has 2 as a factor. Therefore, x must equal 2. The statement is sufficient.

NP

Exponents

Answer: Sufficient

In order for $a^b < 0$, a must be negative. (This is equivalent to saying that $a < 0$.)

If a were nonnegative, then the minimum value a^b could take would be 0, regardless of the value of b.

NP

Is the statement sufficient?

Is x/y even?

1) x and y are both even.

NP

What is the greatest number of primes that could be included in a set composed of four consecutive integers? Name the elements of the set.

NP

Odds & Evens

Answer: Insufficient

Even numbers can be represented as $2m$ or $2n$, where m and n are integers. (Think about why this is.) Dividing would give $(2n)/(2m)$, or just n/m. Not only can it not be determined whether this result is even (e.g., $x = 40$ and $y = 4$) or odd (e.g., $x = 44$ and $y = 4$), we cannot even determine that it will be an integer! (e.g, $x = 42$ and $y = 4$.) The statement is insufficient.

NP

Prime Numbers

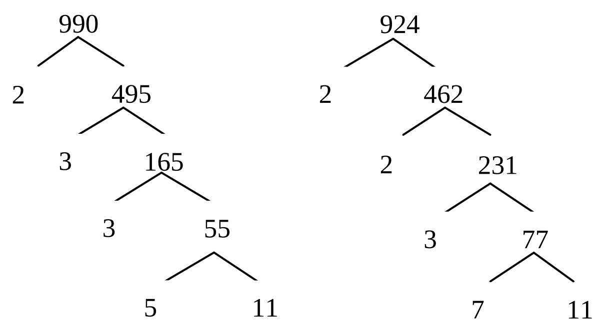
Answer: 3, in the set {2, 3, 4, 5}

Any set composed of four consecutive integers will contain two even and two odd integers. Since 2 is the only even integer that is prime, no such sets can have four primes, and sets that do not contain 2 can have, at most, two primes. The only set with three primes is {2, 3, 4, 5}.

Why isn't {1, 2, 3, 4} acceptable as another solution to this question?

NP

What is the greatest common factor of 990 and 924?



NP

x is divisible by 144. If $\sqrt[3]{x}$ is an integer, then which of the following is $\sqrt[3]{x}$ definitely divisible by? (Choose all that apply)

144

2, 2, 2, 2, 3, 3

a) 4 b) 8 c) 9 d) 12

NP

Greatest Common Factor

Answer: 66

To find the Greatest Common Factor of 2 or more numbers, you need to figure out all the factors they share in common. In this case, 990 and 924 each have one 2, one 3 and one 11. That means that the GCF will be $2 \times 3 \times 11$, or 66.

NP

Divisibility

Answer: a) 4 and d) 12

Remember that when we complete a prime box for a variable, that variable could still have additional factors. For the cube root of a number to be an integer, the original number must have 3 of each prime factor, or some multiple of 3 (3, 6, 9, etc.). In this case, that means the factors of x that we can't see must include at least two additional 2s and one additional 3. From this information, we can definitively conclude that $\sqrt[3]{x}$ must have two 2s and one 3 as factors. 4 and 12 are the only numbers in the list we can guarantee are factors of $\sqrt[3]{x}$.

NP

What is the only two-digit number that is both a perfect square and a perfect cube?

NP Exponents & Roots

Answer: 64

We need a 2-digit integer that is both a perfect square *and* a perfect cube. This set includes all integers of the form $m^3 = n^2$, where both m and n are integers. Manipulating the equation tells us that $n = m^{3/2}$. Thus we can only choose integers for m that will make n an integer—so m must be a perfect square. The only perfect square that works is 4: $4^3 = 64$, a 2-digit integer. 9 doesn't work, because $9^3 = 729$, a 3-digit integer. 1 doesn't work either, because $1^3 = 1$, a 1-digit integer.

NP

Is the statement sufficient?

Is $|x| > |y|$?

1) $x - y > 0$

NP Testing Positive and Negative Cases

Answer: Insufficient

When variables are inside absolute values, a big unknown is whether the variables are positive or negative. If x and y are both positive, then the answer to the question will be yes. But now suppose that x is 3 and y is –7. 3 – (–7) = 10. In this case, the answer to the question is no. We have a yes case and a no case. The statement is insufficient.