EIVs

Factor:

$$x^2 - 11x + 30 = 0$$

EIVs

For each of the following, could the answer be an integer if *x* is an integer greater than 1?

a) $x^{10} + x^{-10} =$

b) $x^{1/6} + x^{1/2} =$

EIVs

Factoring Quadratic Equations

Answer: $(x - 5)(x - 6) = 0$

Since the last sign is positive, set up 2 parentheses with the sign of the middle term.
$(x - \quad)(x - \quad)$
Find two numbers that multiply to 30 and add to 11 and place them in the parentheses.
$(x - 5)(x - 6)$

What values for x solve the equation?

EIVs

Fractional and Negative Exponents

Answer: a) No; b) Yes

a) No. $x^{-10} = 1/x^{10}$. For any $x > 1$, this won't be an integer.

b) Yes. This is equivalent to $\sqrt[6]{x} + \sqrt{x}$, so if x has an integer sixth root this will be an integer. For example, if x equals 64, the sixth root of x is 2, and the square root is 8.

Any number with an integer sixth root will have an integer square root. Why?

EIVs

Is it possible to solve for a single value of x in each of the following systems of equations?

a) $2x + 3y = 8$
$2x - y = 0$

b) $x^2 + y - 17 = 0$
$y = 2x$

c) $2x - 4y = 13$
$-6x + 12y = -39$

EIVs

Basic Equations

Answer: a) Yes; b) No; c) No

a) Yes. We are given 2 linear equations. There are no xy terms or x/y terms.
b) No. There is an x^2 term. Even if $2x$ is substituted into the first equation for y, 17 isn't a perfect square, so we should expect the quadratic to have 2 distinct solutions.
c) No. The two equations are equivalent. The second equation is just the first equation multiplied by –3.

EIVs

What is the 25th term of this sequence?

$S_n = S_{n-1} - 10$ and $S_3 = 0$.

EIVs

Linear Sequences

Answer: –220

First, we need to convert the recursive sequence definition provided into a direct sequence formula. Each term is 10 less than the previous one. Therefore $S_n = -10n + k$, where k is some constant that we must determine. Use S_3 to find a value for k: $0 = -10(3) + k$. Thus, $k = 30$, so $S_n = -10n + 30$. Now we plug in 25 for n: $S_{25} = -10(25) + 30 = -220$.

Alternatively, we could plug in 0 for S_3 and find that $S_4 = -10$, $S_5 = -20$, $S_6 = -30$, etc. Thus, $S_{25} = -220$.

EIVs

Is the statement sufficient?

What are the solutions to the equation $x^2 + kx - 10 = 0$, where k is a constant?

(1) One of the solutions is –5.

EIVs

Is the statement sufficient?

Is $x > y$?

(1) $ax < ay$

EIVs

Factoring Quadratic Equations

Answer: Sufficient

If one solution is –5, we know one of the factors of the quadratic expression is $(x + 5)$. We now know the other factor is $(x - 2)$ because the two numbers in parentheses must multiply to –10. Therefore the other solution is $x = 2$. The statement is sufficient.

EIVs

Positives & Negatives

Answer: Insufficient

We do not know the sign of a, so we cannot simply divide by a on both sides. We must consider two possible scenarios when rephrasing statement (1). If $a > 0$, then we can divide by a on both sides and $x < y$. However, if $a < 0$, after dividing we flip the inequality sign and get $x > y$. The statement is insufficient.

EIVs

Solve for y:

$$y^2 + 7y - 60 = 0$$

EIVs

What is the value of x?

$$5^{3x} = 5^{7x-4}$$

EIVs

Factoring Quadratic Equations

Answer: y = –12, 5

Since the last sign is negative, set up 2 parentheses with opposite signs. $(y + \quad)(y - \quad)$
Find two numbers that multiply to 60 and subtract to 7:
$12 \times 5 = 60 \qquad 12 - 5 = 7$
Place the larger number in the parentheses with the same sign as the middle term $(+7y)$:
$(y + 12)(y - 5) = 0$
If $y + 12 = 0$, then $y = -12$. If $y - 5 = 0$, then $y = 5$.

EIVs

Same Base

Answer: 1

Since the bases are equal, we can simply set the exponents equal to each other.

$$3x = 7x - 4$$
$$4 = 4x$$
$$1 = x$$

EIVs

What is the minimum value of $f(x) = -5 + (x + 7)^2$, and at what value of x does it occur?

EIVs

What are all possible values of x?

$$x^2 - 27x + 50 = 0$$

EIVs

Quadratic Functions

Answer: minimum value = –7, x = –5

The squared expression will always be non-negative, so to make $f(x)$ as small as possible, make the squared expression as small as possible – set it equal to zero. If $x + 7 = 0$, $x = -7$. Once you have the x value, plug it back into the original equation to solve for the minimum value. $f(x) = -5 + (0)^{2.}$ Therefore, the minimum value is –5.

Remember, $f(x)$ and y are synonymous.

EIVs

Factoring Quadratic Equations

Answer: x = 2 or 25

Since the last sign is positive, set up 2 parentheses with the sign of the middle term.
$(x - \quad)(x - \quad)$

Find two numbers that multiply to 50 and add to 27 and place them in the parentheses.
$(x - 2)(x - 25) = 0.$

EIVs

Simplify:

$$-\frac{b}{7} \geq 4$$

EIVs

Simplify:

a) $4^5 + 4^5 + 4^5 + 4^5$

b) $xw + yw + zx + zy$

EIVs

Inequalities

Answer: $b \leq -28$

To isolate b, multiply both sides by -7 and flip the direction of the inequality sign.

When multiplying or dividing an inequality by a negative number, remember to switch the direction of the inequality sign.

EIVs

Factoring

Answer: a) 4^6; b) $(w + z)(x + y)$

a) The greatest common factor is 4^5.
$4^5(1 + 1 + 1 + 1) = 4^5(4) = 4^6$.
Make sure to look for common terms that can be factored out of an expression. Factoring is often a crucial step toward solving an equation.

b) Factor by grouping: $(xw + yw) + (zx + zy) =$
$w(x + y) + z(x + y) = (w + z)(x + y)$.

If you have 4 expressions and 4 variables, look to factor by grouping.

EIVs

Solve for each of the following:

a) If $x = \frac{7-y}{2}$, What is $2x + y$?

b) If $\sqrt{2t+r} = 5$, What is $3r + 6t$?

EIVs

Distribute:

$(b + 7)(b - 10)$

EIVs

MADS Manipulations

Answer: a) 7; b) 75

a) Multiply both sides by 2 and add y to each side.
$2x + y = 7$

b) Square both sides and multiply by 3.
$6t + 3r = 75$

EIVs

FOIL

Answer: $b^2 - 3b - 70$

Use FOIL – First, Outer, Inner, Last

$(b)(b) + (b)(-10) + (7)(b) + (7)(-10)$

$b^2 - 10b + 7b - 70$
$b^2 - 3b - 70$

EIVs

If 2 is one solution to the equation $x^2 - 9x + c = 0$, where c is a constant, what is the other solution?

EIVs

What error has been made?

$$x^2 = 36$$
$$\sqrt{x^2} = \sqrt{36}$$
$$x = 6$$

EIVs

Factoring Quadratic Equations

Answer: 7

Work backwards – even though we do not know the value of c, since 2 is one solution, we know the factored form of the quadratic is
$(x-2)(x-?)$. We also know that the two numbers in parentheses must add to -9. Therefore the factored form is $(x-2)(x-7)$ and the other solution is $x = 7$.

This problem can also be solved by plugging $x = 2$ into the original equation and solving for c.

EIVs

Even Exponents

Answer:

Remember, $\sqrt{x^2} = |x|$. So after we take the square root of both sides, we have $|x| = 6$.

This gives two possibilities: $x = 6$ or $x = -6$.

Alternatively, simply recall that there are always two possible solutions in exponential equations with an even exponent. Thus when $x^2 = 36$, $x = 6$ or -6.

EIVs

If $c < 4$, what is the range of possible values of d for the equation $3c = -6d$?

EIVs

What are the roots of $x^3 - x = 0$?

EIVs

Extreme Values

Answer: $d > -2$

We can actually replace c with its extreme value, which is "less than 4." The equation will read 3(less than 4) = −6d. So (less than 12) = −6d. Divide by −6, and remember to flip the sign, because we're dividing by a negative. Thus we have (greater than −2) = d.

EIVs

Factoring

Answer: $x = 0, -1, \text{or } 1$

Factor the equation, since we already have 0 on one side:

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0, -1, \text{or } 1.$

The temptation is to move x to the other side and divide both sides by x, leaving us with $x^2 = 1$. Avoid dividing away a variable unless you *know* it does not equal 0.

EIVs

Consider the formula $H = \frac{2a^3}{b}$.

If *a* is doubled and *b* is increased by a factor of 4, by what factor is *H* increased?

EIVs

Formulas with Unspecified Amounts

Answer: 2

The exponent of 3 on *a* means when we double *a*, the whole formula gets multiplied by 2^3, or 8. *b* has no exponent, but it is in the denominator, so quadrupling it is the equivalent of multiplying the formula by 1/4. Thus, *H* gets multiplied by 8 × 1/4 = 2.

EIVs

What is *x*? (Hint: Try a method other than substitution)

$x + y = 10$

$3x - 5y = 6$

EIVs

Solving by Combination

Answer: 7

One way to solve for a variable when you have two equations is to combine the equations in a way that eliminates one variable. In this case, we can multiply the first equation by 5, and then add it to the second equation, giving us:

$5x + 5y = 50$
$\underline{3x - 5y = 6}$
$8x + 0y = 56 \rightarrow x = 7$

On the GMAT, combination is often faster than substitution.

EIVs

Is the statement sufficient?

Is $xy < 25$?

(1) x and y are both less than 5.

EIVs

Solve for w:

$2^{2w} = 8^{w-5}$

EIVs

Extreme Values

Answer: Insufficient

We cannot simply multiply $x < 5$ and $y < 5$ to get $xy < 25$. If x and y are both negative, xy could be greater than 25.

Example: $(-10)(-4) = 40$.

Could we multiply $x > 5$ and $y > 5$ to get $xy > 25$?

EIVs

Same Base

Answer: $w = 15$

We must first obtain the same base on both sides. Convert the 8 into a power of 2:

$2^{2w} = (2^3)^{w-5}$ $\quad$ $2^{2w} = 2^{3w-15}$

Now that the bases are equal, we can set the exponents equal to each other:
$2w = 3w - 15 \rightarrow w = 15$.

EIVs

Solve:

$$(x-4)^2 = 49$$

EIVs

The first few steps of a problem are shown. Finish the problem and answer the question: what is *x*?

$$\sqrt{x+3} = x-3$$
$$x+3 = (x-3)^2$$
$$x+3 = x^2 - 6x + 9$$
$$0 = x^2 - 7x + 6$$

EIVs

Taking the Square Root

Answer: $x = 11$ or -3

Do not multiply out $(x-4)^2$ if there is a perfect square on one side of the equation. Instead, take the square root of both sides, and remember to place the side of the equation containing the unknown in an absolute value. $|x-4| = 7$. Our two solutions to this equation are $x - 4 = 7$ and $x - 4 = -7$. Solving these two equations gives us $x = 11$ and -3.

EIVs

Eliminating the Root

Answer: $x = 6$ (x does NOT equal 1!)

Although this equation can be simplified and factored into $(x-6)(x-1)=0$, you need to be careful. When you square an equation containing a variable, you may create extraneous solutions. Potential answers need to be plugged back in to the original equation and verified. 6 is a genuine solution, 1 is not.

Try plugging 1 back into the original equation to verify that x cannot equal 1.

EIVs

What is $x + y + z$?

$$x + y = 8$$
$$x + z = 11$$
$$y + z = 7$$

EIVs

Simplify:

$$(\sqrt{2} + 3)(\sqrt{2} - 3)(2 - \sqrt{3})(2 + \sqrt{3})$$

EIVs

Solving by Combination

Answer: 13

There is often a faster method than solving for the value of each variable. In this case, we can simply add all the equations together!

$$\begin{array}{rcl} x + y & & = 8 \\ x & + z & = 11 \\ \underline{\quad y} & \underline{+ z} & \underline{= 7} \\ 2x + 2y & + 2z & = 26 \\ x + y & + z & = 13 \end{array}$$

Remember, $x + y + z$ is a "combo." In this type of problem there is a good chance you will not need to determine the individual values of the variables.

EIVs

Special Products

Answer: −7

Remember, $(a + b)(a - b) = a^2 - b^2$.

Therefore, our expression is equal to:
$(2 - 9) \times (4 - 3) = (-7)(1) = -7$

EIVs

Is the statement sufficient?

A group of rabbits multiplies at a constant rate. By what factor does its population increase every day?

(1) The population grows from 200 to 5,000 in one week.

EIVs

Simplify:

$$\frac{a-b}{\sqrt{a}+\sqrt{b}}$$

EIVs

Population Growth

Answer: Sufficient

Remember, we just need to know that we can calculate the rate of growth. They've given us the initial and final numbers of rabbits, as well as the time span. That is enough to calculate the rate of growth. For example, in 7 days, the population increases by a factor of 5,000/200 = 25. In one day it increases by a factor of $\sqrt[7]{25}$. (We do not, however, need to actually do this calculation on a Data Sufficiency question!)

EIVs

Using the Conjugate

Answer: $\sqrt{a}-\sqrt{b}$

Anytime there is a square root term in the denominator that is added to or subtracted from another term we can multiply by the conjugate (the same expression, but with the sign on the 2nd term flipped) to simplify:

$$\frac{a-b}{\sqrt{a}+\sqrt{b}}\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)=\frac{(a-b)\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)}=\frac{(a-b)\left(\sqrt{a}-\sqrt{b}\right)}{(a-b)}=\sqrt{a}-\sqrt{b}$$

Alternatively, you could use the special product $a^2 - b^2 = (a + b)(a - b)$ to solve. In this case, $a-b=\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$, and so the term $\sqrt{a}+\sqrt{b}$ would cancel from the top and bottom, leaving $\sqrt{a}-\sqrt{b}$

EIVs

Are the two statements sufficient when combined?

What is x?

(1) $\frac{3x}{3y+5z} = 8$ (2) $6y + 10z = 18$

EIVs

Simplify:

$$\frac{3}{2+\sqrt{3}}$$

EIVs

Combo Problems

Answer: Sufficient

Divide the equation in (2) by 2 and get $3y + 5z = 9$. Substitute 9 for the denominator of the fraction in (1). This leaves an equation with one variable, x.

Remember, when you see 3 variables and only 2 equations, you should not automatically assume that you cannot solve for a particular value.

EIVs

Using the Conjugate

Answer: $6 - 3\sqrt{3}$

To remove a square root from a denominator of the form $a + \sqrt{b}$, multiply the fraction by $\frac{a-\sqrt{b}}{a-\sqrt{b}}$. The form is the same whether you are dealing with numbers, variables, or a combination of the two.

$$\frac{3}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = \frac{3\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)} = \frac{6-3\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3} =$$

$$\frac{6-3\sqrt{3}}{1} = 6-3\sqrt{3}.$$

EIVs

Is the statement sufficient?

In a sequence of terms in which each term is twenty-three times the previous term, what is the 11th term?

(1) The 19th term is 40.

EIVs

How would you factor each of the following expressions?

a) $x^5 - x^3$

b) $4^8 + 4^9 + 4^{10}$

c) $m^{n-2} - 3m^n + 4m^{n+1}$

EIVs

Exponential Sequences

Answer: Sufficient

We could simply work backwards from the 19th term, dividing each term by 23. $S_{18} = 40/23$, $S_{17} = 40/23^2$, etc.

Generally, in a sequence, if you know the factor that each term is being multiplied by, 23 in this case, and if you know just one term, it is sufficient to solve for any other term in the sequence.

EIVs

Factoring

Answer:

a) The GCF is x^3, the smaller power.
$x^3(x^2 - 1) = x^3(x + 1)(x - 1)$.

b) The GCF is 4^8. $4^8(1 + 4^1 + 4^2) = 4^8(21)$.

c) The smallest power of m is the GCF. Here it is m^{n-2}:
$m^{n-2}(1 - 3m^2 + 4m^3)$.

EIVs

Solve by picking numbers:

Bottle 1, with capacity *x*, is half full of water. Bottle 2, with capacity *y,* is one sixth full. If Bottle 2 is three times the size of Bottle 1 and the contents of Bottle 1 are emptied into Bottle 2, how many liters of water, in terms of *y*, are in Bottle 2?

a) $\frac{1}{2}y$ b) $\frac{1}{6}y$ c) $\frac{2}{3}y$ d) $\frac{1}{3}y$ e) $\frac{5}{6}y$

EIVs

Set up an appropriate equation to describe the given scenario:

The elasticity (*e*) of a material is directly proportional to the square of its density (*d*) and inversely proportional to the cube of its mass (*m*).

EIVs

Picking Numbers

Answer: d) 1/3 *y*

When problems involve many fractions and no specific quantities, it is best to pick numbers that are multiples of all the denominators in the problem. The least common multiple of 6 and 2 is 6. Thus, let the capacity of Bottle 1 = 6 and the capacity of Bottle 2 = 18. Bottle 1 holds 3 liters and bottle 2 holds 3 liters. Bottle 1 is dumped into Bottle 2, which then contains 6 liters. Test each answer choice with *y* = 18 and notice that d) is the solution, since $\frac{1}{3}(18) = 6$.

EIVs

Direct/Inverse Proportionality

Answer: $e = \frac{kd^2}{m^3}$

A constant k is used in expressions of direct or inverse proportionality. e is directly proportional to d^2, which means $e = kd^2$. e is also inversely proportional to m^3, so $e = k/m^3$. Putting these two equations together, we get $e = \frac{kd^2}{m^3}$.

Note that k in the final equation must be the product of the k constants in the first two equations, but since k could be any value, we can repeat the use of k for simplicity.

EIVs

Is the statement sufficient?

Given that $x^2 - y^2 = 20$, what is y?

(1) $x + y = 5$

EIVs

Identify the error:

$8! + 2 \leq x \leq 8! + 10$ implies that $2 \leq x \leq 10$.

EIVs

Special Products

Answer: Sufficient

Factor the special product. We know that $(x + y)(x - y) = 20$. Since $(x + y) = 5$, $(x - y) = 4$. We have two linear equations, so we know we can solve for x and y individually.

The equations are linear because there are no squared terms, no xy terms, and no x/y terms. The solutions are $x = 4.5$, $y = 0.5$.

EIVs

Compound Inequalities

Answer: $2 \leq x \leq 10$ is incorrect

In a compound inequality, you must perform the same operation to *all 3 expressions*, not just the outside expressions. If you subtract 8! from all 3 expressions, you get $2 \leq x - 8! \leq 10$.

EIVs

Is the statement sufficient?

Is $xy < 0$?

(1) $xz > 0$ and $yz < 0$

EIVs

Factor:

$$\frac{x^2}{9} - 25y^2$$

EIVs

Positives & Negatives

Answer: Sufficient

$xz > 0$ means x and z have the <u>same sign</u>. $yz < 0$ means y and z have <u>opposite signs.</u> Together, this means that x and y must have opposite signs and consequently $xy < 0$.

EIVs

Special Products

Answer: $\left(\frac{x}{3} + 5y\right)\left(\frac{x}{3} - 5y\right)$

This expression is a slightly more complicated version of the special product $a^2 - b^2 = (a + b)(a - b)$. Notice that x^2, 9, 25, and y^2 are all perfect squares:

$$\frac{x^2}{9} - 25y^2 = \left(\frac{x}{3}\right)^2 - (5y)^2 = \left(\frac{x}{3} + 5y\right)\left(\frac{x}{3} - 5y\right)$$

EIVs

The first three terms of a linear sequence are –2, 18, and 38. What is the rule for this sequence?

EIVs

If $10 \leqslant m \leqslant 20$ and $-2 \leq p \leq 15$, and *m* and *p* are both integers, what is the maximum possible value for *m* – *p*?

EIVs

Linear Sequences

Answer:

Since the terms are increasing by 20, we know the rule is $S_n = 20n + k$. Use any of the three given terms to solve for *k*:

$S_2 = 20(2) + k$ $\qquad$ $18 = 40 + k$

$k = -22$

The rule is $S_n = 20n - 22$.

Try solving for *k* using S_1 or S_3 and verify that you get the same value. Could you express this sequence using a recursive definition?

EIVs

Extreme Values

Answer: 22

To maximize *m* – *p*, make *m* as large as possible and make *p* as small as possible. $m = 20$ and $p = -2$. $20 - (-2) = 22$.

EIVs

Is the statement sufficient?

What are the solutions to $x^2 - 10x + b = 0$?

1) The sum of the roots is 10.

EIVs

Solve by picking numbers and calculating a target:

What is the average of $(x + y)^2$ and $(x - y)^2$?

a) $2x^2 - 2y^2$
b) $x^2 + 4xy + y^2$
c) $x^2 + y^2$

EIVs

Factoring Quadratic Equations

Answer: Insufficient

Since the middle term of the quadratic expression is $-10x$, we know the factored form would take the form $(x - a)(x - b)$, where $a + b = 10$. Thus we already knew the sum of the roots is equal to 10 before statement (1), so it is not enough information.

EIVs

Picking Numbers

Answer: c)

Let's pick $x = 3$ and $y = 2$.

$(3 + 2)^2 = 25$ and $(3 - 2)^2 = 1$, so the average is $\frac{25 + 1}{2} = 13$.

Now, let's test each answer choice:
a) $2(3)^2 - 2(2)^2 = 10$
b) $(3)^2 + 4(3)(2) + (2)^2 = 37$
c) $(3)^2 + (2)^2 = 13$