FOUNDATIONS OF GMAT MATH

Math Strategy Guide

This supplemental guide provides in-depth and easy-to-follow explanations of the fundamental math skills necessary for a strong performance on the GMAT.

Foundations GMAT of Math Strategy Guide, Fourth Edition

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This Foundations of GMAT Math Guide is a supplement to our

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Dear Student,

Thank you for picking up this study guide for the GMAT—we hope that it refreshes your memory of the junior-high math that you haven't used in a long time.

This book got its start a couple of years ago with one of our Los Angeles Instructors, Mike Kim. Mike had a number of private students who had not seen many of the ground-level math concepts in quite a while. Mike, being a very productive guy, took it upon himself to create a set of presentations that would give his students the runway they needed to get back up to speed on algebra, geometry, and all of the other topics tested on the GMAT. The Foundations of Math Workshops were such a success that we decided to turn Mike's work into this book.

Many people contributed to the Foundations of GMAT Math book. First is David Mahler, who tirelessly found new and better ways to explain and present various concepts. Next is Robert Wilburn, who lent a hand with several chapters. Chris Ryan, the Company's Lead Instructor and Director of Curriculum Development, provided an editorial eye to the proceedings. And last, Dan McNaney made sure that all of the words, figures and drawings were translated properly to the printed page.

At Manhattan GMAT, we continually aspire to provide the best Instructors and resources possible. We hope that you'll find our dedication manifest in this book. If you have any comments or questions, please e-mail me at andrew.yang@manhattangmat.com. I'll be sure that your comments reach the rest of the team —and I'll read them too.

Best of luck in preparing for the GMAT!

Sincerely,

Andrew Yang
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TABLE OF CONTENTS

Introduction

to

FOUNDATIONS OF GMAT MATH

INTRODUCTION

Welcome to Manhattan GMAT's Foundations of GMAT Math guide!

If you've decided to take the GMAT, but are feeling a little overwhelmed by the quantitative section, you are not alone! It's likely been a while since you've seen some (or most) of these concepts. The purpose of this book is to help you relearn basic GMAT math and become proficient with a number of core math skills that are necessary to score well on the GMAT.

We'll cover a variety of topics, including

- Order of operations
- Equations with variables in them
- Combining equations
- Translating word problems into equations
- Inequalities and absolute values
- Divisibility and exponents
- Fractions, decimals, and percents
- Geometry

Perhaps you're not familiar with all of the terms above. Don't worry—you'll be quite comfortable with them by the end of this book.

Helpful Hints for Using This Book

Notes Used in this Book

Time Saving Tips suggest ways that you can speed up your work.

Safety Tips warn you about common traps and errors.

Nerd Notes give you some extra facts that you might find interesting as you learn more about mathematics.

Also, if there's a word we use that you don't understand, flip back to the glossary—you'll likely find a definition there.

Drill Sets

Our goal is not only to introduce and review fundamental math skills, but also to provide a means for you to practice applying these skills.

Toward this end, we have included a number of drill sets throughout the book. After each important topic there is a set of questions entitled "Check Your Skills". **If you find these questions challenging, it's probably a sign that you need to re-read the section you just finished.**

At the end of each chapter, you'll find drill sets that cover everything you learned in the entire chapter. These drills offer a great way to reinforce the content and make sure that you really understand how to apply the material.

How Does this Book Relate to the Other Manhattan GMAT Strategy Guides?

Manhattan GMAT publishes eight Strategy Guides that form the core of its curriculum. This Foundations of GMAT Math Guide is intended to complement Manhattan GMAT's 5 math Strategy Guides by providing a resource for more in-depth instruction in a few foundational areas throughout the curriculum.

5 Math Strategy Guides

1. Number Properties
2. Fractions, Decimals, & Percents
3. Equations, Inequalities, & VICs
4. Word Translations
5. Geometry

Foundations of GMAT Math

In-depth explanation of fundamental math topics and skills

As you work through this book, many of the concepts and techniques may be familiar to you. Feel free to skim the topics that you feel comfortable with and spend more time on other chapters. One good way to determine if you understand a chapter is to see if you can quickly perform the Check Your Skills problems distributed throughout.

Chapter 1

of

FOUNDATIONS OF GMAT MATH

EQUATIONS:
SOLVING FOR VARIABLES

In This Chapter . . .

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- The Order of Operations (PEMDAS)
- Solving for a Variable With One Equation
    - Expressions vs. Equations
    - Isolating a Variable
    - Equation Clean-Up Moves
- Solving for Variables with Two Equations
    - Substitution
- Appendix
    - Multiplication Table
    - Multiplying Larger Numbers
    - Long Division

EQUATIONS: SOLVING FOR VARIABLES

In This Chapter:

- Simplifying expressions, using the order of operations
- Solving an equation with a variable in it
- Combining multiple equations to solve for a variable (or variables)

Equations are the heart and soul of GMAT math. For almost every GMAT math problem, you will have to solve an equation. If you haven't faced equations since you were last in school, this can be intimidating. In this chapter, our objective is to help you become comfortable setting up and solving equations. We'll start with some basic equations (we'll even take out the variables at first), and then work our way up to some pretty tricky problems. Let's dive in.

The Order of Operations (PEMDAS)

$$3 + 4(5 - 1) - 3^2 \times 2 = ?$$

Nerd Note: Technically, an *equation* must have an equals sign. When there are variables and/or numbers, but no equals sign, you are looking at an *expression*.

Before we start dealing with variables, let's spend a moment looking at expressions that are made up of only numbers, such as the example above. It's a string of numbers, with mathematical symbols in between them. Which part of the expression should you focus on first?

Intuitively, most of us think of going in the direction we read, from left to right. When we read a book, moving left to right is a wise move (unless you're reading a language like Chinese or Hebrew). However, when we perform basic arithmetic, there is an order that is of greater importance: **the order of operations**.

The order in which you perform the mathematical functions should primarily be determined by the functions themselves. In the correct order, the six operations are **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, and **A**ddition/**S**ubtraction (or **PEMDAS**).

Before we solve a problem that requires PEMDAS, here's a quick review of the basic operations.

Safety Tip: We will be discussing exponents and roots in more detail in Chapter 5.

Parentheses can be written as ( ) or [ ] or even { }.

Exponents are $5^{\mathbf{2}} \leftarrow$ these numbers. 5^2 ("five squared") can be expressed as 5×5. In other words it is 5 times itself twice, or 2 times.

Likewise, 4^3 ("four cubed," or "four to the third power") can be expressed as $4 \times 4 \times 4$ (4 times itself 3 times).

Roots are very closely related to exponents. $\sqrt[3]{64}$ is the third root of 64 (commonly called the cube root). $\sqrt[3]{64}$ is basically asking the question "What multiplied by itself 3 times equals 64?" $4 \times 4 \times 4 = 64$, so $\sqrt[3]{64} = 4$. The plain old square root $\sqrt{9}$ can be thought of as $\sqrt[2]{9}$. What times itself equals 9? $3 \times 3 = 9$ so $\sqrt{9} = 3$.

Exponents and roots can also undo each other. $\sqrt{5^2} = 5$ and $(\sqrt[3]{7^3}) = 7$.

Multiplication and Division can also undo each other. $2 \times 3 \div 3 = 2$ and $10 \div 5 \times 5 = 10$.

Nerd Note: All of these rules apply to variables as well as numbers (i.e. $y \times 3 \div 3 = y$).

Multiplication can be expressed with parentheses: $(5)(4) = 5 \times 4 = 20$.
Division can be expressed with a division sign (÷), a slash (/) or a fraction bar(—): $20 \div 5 = 20/5 = \frac{20}{5} = 4$. Also remember that multiplying or dividing by a negative number flips the sign:

$4 \times (-2) = -8$ $\qquad$ $-8 \div (-2) = 4$

To review how to multiply larger numbers and to do long division, see the Appendix to this chapter on page 45.

Addition and Subtraction can also undo each other. $8 + 7 - 7 = 8$ and $15 - 6 + 6 = 15$

PEMDAS is a useful acronym you can use to remember the order in which operations should be performed. Some people find it useful to write PEMDAS like this:

$$\overrightarrow{\text{PE}^{\text{M}}/_{\text{D}}{}^{\text{A}}/_{\text{S}}}$$

The reason that Multiplication and Division are at the same level of importance is that any Multiplication can be expressed as Division, and vice-versa. $7 \div 2$ is equivalent to $7 \times 1/2$. In a sense, Multiplication and Division are two sides of the same coin.

Addition and Subtraction have this same relationship. $3 - 4$ is equivalent to $3 + (-4)$. The correct order of steps to simplify this expression is as follows:

	$3 + 4(5 - 1) - 3^2 \times 2$
Parentheses	$3 + 4(4) - 3^2 \times 2$
Exponents	$3 + 4(4) - 9 \times 2$
Multiplication or **D**ivision	$3 + 16 - 18$
Addition or **S**ubtraction	$3 + 16 - 18 = 19 - 18 = 1$

Safety Tip: With the two pairs, Multiplication/Division and Addition/Subtraction, do not first solve all the additions and then all the subtractions. Instead, consider them simultaneously as you move from inside parentheses to outside and from **left to right**.

Remember: If you have two operations of equal importance, you should do them in left-to-right order: $3 - 2 + 3 = 1 + 3 = 4$. The only instance in which you would override this order is when the operations are in parentheses: $3 - (2 + 3) = 3 - (5) = -2$.

Let's do two problems together. Try it first on your own, then we'll go through it together:

$$5 - 3 \times 4^3 \div (7 - 1)$$

P

E

M/D

A/S

> **Safety Tip:** Re-write the expression as you simplify it. It might take a bit longer but you're much less likely to make errors that require you to do the whole problem again.

Your work should have looked like this:

$$5 - 3 \times 4^3 \div (\underbrace{7 - 1})$$

$$5 - 3 \times \underbrace{4^3} \div 6 \longrightarrow 4^3 = 4 \times 4 \times 4 = 64 \longrightarrow \begin{array}{r} {}^{2} \\ 16 \\ \times\ 4 \\ \hline 64 \end{array}$$

$$5 - \underbrace{3 \times 64} \div 6 \longrightarrow \begin{array}{r} {}^{1} \\ 64 \\ \times\ 3 \\ \hline 192 \end{array}$$

$$5 - \underbrace{192 \div 6} \longrightarrow \begin{array}{r} 32 \\ 6\overline{)192} \\ -\underline{18} \\ 12 \\ -\underline{12} \\ 0 \end{array}$$

$$\underbrace{5 - 32}$$

$$-27$$

> **Safety Tip:** If you had trouble with the last step (5 – 32), you may want to review integer addition and subtraction. This guide will assume that you are comfortable with basic arithmetic operations.

Let's try one more:

$$32 \div 2^4 \times (5 - 3^2)$$

P

E

M/D

A/S

Here's the work you should have done:

$$32 \div 2^4 \times (5 - 3^2)$$

$$32 \div 2^4 \times (5 - 9)$$

$$32 \div 2^4 \times (-4)$$

$$32 \div 16 \times (-4)$$

$$2 \times (-4)$$

$$-8$$

Safety Tip: Order of operations also applies inside the parentheses.

Check Your Skills

Evaluate the following expressions.

1. $-4 + 12/3 =$
2. $(5 - 8) \times 10 - 7 =$
3. $-3 \times 12 \div 4 \times 8 + (4 - 6) =$
4. $2^4 \times (8 \div 2 - 1)/(9 - 3) =$

Answers can be found on page 29.

Solving for a Variable With One Equation

Expressions vs. Equations

So far, we've been dealing only with expressions. Now we're going to be dealing with equations. The big difference between expressions and equations is that, while expressions only have numbers and/or variables on one side of the equals sign, equations have numbers and/or variables on *both* sides of the equals sign.

Pretty much everything we will be doing with equations is related to one basic principle: we can do anything we want to one side of the equation, *as long as we also do the same thing to the other side of the equation*. Take the equation $3 + 5 = 8$. I want to subtract 5 from the left side of my equation, but I still want my equation to be true. All I have to do is subtract 5 from the right side as well, and I can be confident that my new equation will still be valid.

$$\begin{array}{rcl} 3 + 5 &=& 8 \\ -5 & & -5 \\ \hline 3 \quad &=& 3 \end{array}$$

Note that this would also work if I had variables in my equation:

$$\begin{array}{rcl} x + 5 &=& 8 \\ -5 & & -5 \\ \hline x \quad &=& 3 \end{array}$$

Next we're going to see some of the many ways we can apply this principle to solving algebra problems.

Solving Equations

But first, what does it mean to solve an equation? What are we really doing when we manipulate algebraic equations?

A solution to an equation is a number that, when substituted in for the value of a variable, makes the equation *true*.

Take the equation $2x + 7 = 15$. We are looking for the value of x that will make this equation true. What if we plugged in 3 for x? If we replaced x with the number 3, we would get $2(3) + 7 = 15$. This equation can be simplified to $6 + 7 = 15$, which further simplifies to $13 = 15$. 13 definitely does NOT equal 15, so when $x = 3$, the equation is NOT true. So $x = 3$ is NOT a solution to the equation.

Now, if we replaced x with the number 4, we would get $2(4) + 7 = 15$. This equation can be simplified to $8 + 7 = 15$. Simplify it further, and you get $15 = 15$, which is a true statement.

That means that when $x = 4$, the equation is true. So $x = 4$ is a solution to the equation.

Now the question becomes, what is the best way to find these solutions? What is an efficient way to determine what value or values of a variable will make an equation true? If we had to use trial and error, or guessing, the process could take a very long time. The following sections will talk about the ways in which we can efficiently and accurately manipulate equations so that solutions become easier to find.

Isolating a Variable

Now that we know that we're allowed to change one side of an equation, as long as we make the same change to the other side, let's look at the different kinds of changes we can make. We'll discuss these changes as we try to solve the following problem.

If $5(x-1)^3 - 30 = 10$, then $x = ?$

To solve for a variable, we need to get it by itself on one side of the equals sign. To do that, we'll need to make a number of changes to the equation that will change its appearance, but not its value. The good news is, all of the changes we will need to make to this equation to solve for x will actually be very familiar to you—they're the PEMDAS operations!

To get x by itself, we want to move everything that isn't the variable to the other side of the equation. The easiest thing to move at this stage is the 30, so let's start there. If 30 is being subtracted on the left side of the equation, and we want to move it to the other side, then we need to do the opposite operation in order to cancel it out. So we're going to **add** 30 to both sides, like this:

$$\begin{array}{rcl} 5(x-1)^3 - \cancel{30} & = & 10 \\ \underline{\ +\cancel{30}} & & \underline{+30} \\ 5(x-1)^3 & = & 40 \end{array}$$

Now we've only got one term on the left side of the equation. x is still inside the parentheses, and the parentheses is being multiplied by 5, so the next step will be to move that 5 over to the other side of the equation. Once again, we want to perform the opposite operation, so we'll **divide** both sides of the equation by 5.

$$\frac{\cancel{5}(x-1)^3}{\cancel{5}} = \frac{40}{5} \quad \longleftarrow \text{ These horizontal lines mean division.}$$

$$(x-1)^3 = 8$$

Safety Tip: Whenever you are making these kinds of changes to equations, make sure that you perform an operation on the entire side of the equation, and not on specific terms.

Now at this point we could cube $(x-1)$, but that is going to involve a whole lot of multiplication. Instead, we can get rid of the exponent by performing the opposite operation. The opposite of exponents is roots. So if the left side of the equation is raised to the third power, we can undo that by taking the third root of both sides, also known as the cube root.

$$\sqrt[3]{(x-1)^3} = \sqrt[3]{8}$$

$$(x-1) = 2$$

Safety Tip: The GMAT often uses numbers that make calculations easy. $\sqrt[3]{8}$, for instance, is 2 because $2 \times 2 \times 2 = 8$.

Now that nothing else is being done to the parentheses, we can just get rid of them, so really, the equation is:

$$x - 1 = 2$$

After that, we add 1 to both sides, and we get $x = 3$. This would have been hard to guess!

Now, take a look at the steps that we took in order to isolate x. Notice anything? We **added** 30, then we **divided** by 5, then we got rid of the **exponent** and then we simplified our **parentheses**. We did PEMDAS backwards! And in fact, when you're isolating a variable, it turns out that the simplest way to do so is to reverse the order of PEMDAS when deciding what order you will perform your operations. Start with addition/subtraction, then multiplication/division, then exponents, and finish with terms in parentheses.

Now that you know the best way to isolate a variable, let's go through one more example. Try it on your own first, then we'll go through it together.

If $4\sqrt{(x-6)} + 7 = 19$, then $x = ?$

A/S

M/D

E

P

Let's get started. The equation we're simplifying is $4\sqrt{(x-6)} + 7 = 19$. If there's anything to add or subtract, that will be the easiest first step. There is, so the first thing we want to do is get rid of the 7 by subtracting 7 from both sides.

$$\begin{array}{r} 4\sqrt{(x-6)}+7=19 \\ -7 \quad -7 \\ \hline =12 \end{array}$$

Now we want to see if there's anything being multiplied or divided by the term containing an x. The square root that contains the x is being multiplied by 4, so our next step will be to get rid of the 4. We can do that by dividing both sides of the equation by 4.

$$\frac{\cancel{4}\sqrt{(x-6)}}{\cancel{4}}=\frac{12}{4}$$
$$\sqrt{(x-6)}=3$$

Now that we've taken care of multiplication and division, it's time to check for exponents. And that really means we need to check for exponents and roots, because they're so intimately related. There are no exponents in the equation, but the x is inside a square root, so that's the next thing we need to deal with. In order to cancel out a root, we can use an exponent. Squaring a square root will cancel it out, so our next step is to square both sides.

$$\sqrt{(x-6)}=3$$
$$(\sqrt{(x-6)})^2=(3)^2$$
$$x-6=9$$

Safety Tip: If there are multiple terms inside a square root (e.g. $x – 6$), there may not be parentheses around them, but you should treat them as if there are parentheses when dealing with order of operations.

The final step is to add 6 to both sides, and we end up with $x = 15$.

Check Your Skills

Solve for x in the following equations:

5. $3(x+4)^3-5=19$

6. $\frac{3x-7}{2}+20=6$

7. $\sqrt[3]{(x+5)}-7=-8$

Answers can be found on page 29.

Equation Clean-Up Moves

We've covered the basic operations that we'll be dealing with when solving equations. But what would you do if you were asked to solve for x in the following equation?

$$\frac{5x-3(4-x)}{2x}=10$$

Now x appears in multiple parts of the equation, and our job has become more complicated. In addition to our PEMDAS operations, we also need to be able to simplify, or clean up, our equation. Let's see the different ways we can clean up this equation. First, notice how we have an x in the denominator (the bottom of the fraction) on the left side of the equation.

Check Your Skills Answer Key:

1.	$-4 + 12/3 =$	Divide first
	$-4 + 4 = 0$	Then add the two numbers
	Answer: 0	
2.	$(5 - 8) \times 10 - 7 =$	
	$(-3) \times 10 - 7 =$	First, combine what is inside the parentheses
	$-30 - 7 =$	Then multiply −3 and 10
	$-30 - 7 = -37$	Subtract the two numbers
	Answer: −37	
3.	$-3 \times 12 \div 4 \times 8 + (4 - 6)$	
	$-3 \times 12 \div 4 \times 8 + (-2)$	First, combine what's in the parentheses
	$-36 \div 4 \times 8 + (-2)$	Multiply −3 and 12
	$-9 \times 8 - 2$	Divide −36 by 4
	$-72 + (-2) = -74$	Multiply −9 by 8 and subtract 2
	Answer: −74	
4.	$2^4 \times (8 \div 2 - 1) / (9 - 3) =$	
	$2^4 \times (4 - 1)/(6) =$	$8/2 = 4$ and $9 - 3 = 6$
	$16 \times (3)/(6) =$	$4 - 1 = 3$ and $2^4 = 16$
	$48/6 =$	Multiply 16 by 3
	$48/6 = 8$	Divide 48 by 6
	Answer: 8	
5.	$3(x + 4)^3 - 5 = 19$	
	$3(x + 4)^3 = 24$	Add 5 to both sides
	$(x + 4)^3 = 8$	Divide both sides by 3
	$(x + 4) = 2$	Take the cube root of both sides
	$x = -2$	Remove the parentheses, subtract 4 from both sides
	Answer: $x = -2$	
6.	$\frac{3x-7}{2}+20=6$	
	$\frac{3x-7}{2}=-14$	Subtract 20 from both sides
	$3x - 7 = -28$	Multiply both sides by 2
	$3x = -21$	Add 7 to both sides
	$x = -7$	Divide both sides by 3
	Answer: $x = -7$	
7.	$\sqrt[3]{(x+5)}-7=-8$	
	$\sqrt[3]{(x+5)}=-1$	Add 7 to both sides
	$x + 5 = -1$	Cube both sides, remove parentheses
	$x = -6$	Subtract 5 from both sides
	Answer: $x = -6$	